SPIRITUAL STUDY GROUP
(SSG)

आध्यात्मिक अध्ययन संस्था
(आऽस)

★ ॐ श्रीः ★

VEDIC MATHEMATICS

OR

SIXTEEN SIMPLE MATHEMATICAL FORMULAE FROM THE VEDAS

(For One Line Answers to all Mathematical Problems)

-: वैदिक गणित सूत्र :-

१ एकाधिकेन पूर्वेण
२ निखिलं नवतश्चरमं दशतः
३ ऊर्ध्वतिर्यग्भ्याम्
४ परावर्त्य योजयेत्
५ शून्यं साम्यसमुच्चये
६ (आनुरूप्ये) शून्यमन्यत्
७ संकलनव्यवकलनाभ्याम्
८ पूरणपूरणाभ्याम्
९ चलनकलनाभ्याम्
१० यावदूनम्
११ व्यष्टिसमष्टिः
१२ शेषाण्यङ्केन चरमेण
१३ सोपान्त्यद्वयमन्त्यम्
१४ एकन्यूनेन पूर्वेण
१५ गुणितसमुच्चयः
१६ गुणकसमुच्चयः

L 3-4 E

वेदात् सर्वं प्रसिद्ध्यति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं ।

Vedas are the Eternal Source of all Knowledge !

R
510
PUR-V:8

R
510
PUR-V:8

5900

गुरुकुल कांगड़ी विश्वविद्यालय
कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

5980

R पुस्तकालय
गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार

वर्ग संख्या 510 आगत संख्या 5980
PUR-V:8

पुस्तक—वितरण की तिथि नीचे अंकित है। इस तिथि सहित २० वें दिन तक यह पुस्तक पुस्तकालय में वापिस आ जानी चाहिए। अन्यथा १० पैसे के हिसाब से विलम्ब—दण्ड लगेगा।

5980

आध्यात्मिक अध्ययन संस्था (आअस)

SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE-247 667 (INDIA)

वैदिक गणित अध्ययन

5980

पुस्तकालय
पुस्तकालय
कांग

MOST CORDIAL WELCOME

The essence of the Vedic system is that it is simple, direct, one-line and mental. The formulae on which it is based are given in word form, which renders them applicable in a wide variety of situations. They are easy to remember, easy to understand and a delight to use.

The Vedic system does not have just one method of solving a particular problem or doing a sum; there is always a general method, and there are various other methods which can be used if one wishes, or if the sum is recognised as being of a special type. This element of choice in the Vedic system, and even of innovation, together with the mental approach, brings a new dimension to the study and practice of mathematics. The variety and simplicity of the methods brings fun and amusement, the mental practice leads to a more agile, alert and intelligent mind, and innovation naturally follows.

Numbers themselves have no existance in the physical world, only applications; so also geometry and algebra. The numbers exist in the mind, which loves simplicity and pattern, and it is the common underlying patterns and relationships, which embody the basic laws of mathematics, that constitute the Vedic system.

PROGRAMME

	Prayer
17.30	Welcome and Introduction of Chief Guest Dr. G.C. Nayak
17.35	Inaugural address Dr. A.S. Arya
17.45	Introduction of Keynote Speaker Dr. S.N. Pandey
17.50	Keynote Address
18.30	Vote of Thanks Prayer

- 16 books on Vedic Maths. available
- 6 months correspondence course in Hindi & English available

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
Vedas are the Eternal Sources of All Knowledge

R
510
PUR-V:8

आध्यात्मिक अध्ययन संस्था (आअसं)
वैदिक विज्ञान संस्थान

Spiritual Study Group (SSG)
INSTITUTE OF VEDIC SCIENCES
41/2, Amod Kunj
University Campus
ROORKEE-247667 (INDIA)

वैदिक गणित अध्ययन

VEDIC MATHEMATICS

The Sutras (aphorisms) apply to and cover each and every chapter of each and every branch of mathematics (including arithmetic, algebra. geometry - plane and solid, trigonometry - plane and spherical, conics - geometrical and analytical, astronomy, calculus - differential and integral etc., etc. In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction;

The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word " mental ";

Swami Bharati Krishen Tirth
Maharaj ji

This Workshop consisted of two parts. 55 participants registered for the Elementary Vedic Mathematics sessions. There were 60 registerations for the Advanced course.

Futher nearly 25 persons attended most of the sessions without registration.

The response forms (copy enclosed) were given to all the participants. The summary of their response is as given below.

Q.No .	Yes (%)	No. (%)	No Comments (%)
1	100%	-	-
2	97.5%	-	2.5%
3	92.5%	2.5%	5.0%
4	95.0%	2.5%	2.5%
5	87.5%	2.5%	10.0%
6	87.5%	7.5%	5.0%
7	82.5%	10.0%	7.5%
8	90%	5.0%	5.0%
9	87.5%	2.5%	10.0%
10	87.5%	-	12.5%
11	90.0%	-	10.0%

OPINIONS OF PARTICIPANTS

- This is the most interesting and thrilling time of life when I came in contact with the Vedic Mathematics it gave such a pleasure so that I can't explain it in words. Being a Mathematics teacher I have interest in maths but this Vedic Mathematics made me so thrilling it is a golden time for me. Now I am thankful to the convenor and Prof. Nicholas who gave me such an excellent chance.

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

वैदिक गणित अध्ययन

Everybody who has learnt a little about the Vedic Mathematics has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics.

The Spiritual Study Group at Roorkee has been organising, Elementary and Advanced Courses on Vedic Mathematics since May, 85. All the participants are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked. The minimum score in favour of Vedic Mathematics for any question in any course sofar was 80%. Most of the questions got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Mathematics.

A few glimpses of the participants opinions :

- A boon for all competitive examinations.
- Could never imagine it would be so interesting.
- Very pleasant to have this course at the university.
- How advanced our ancestors were even in Mathematics.
- If Vedic Maths was taught in every School, computers would have developed in India.
- Excellent course.
- Want to learn advanced topics.
- Increases devotion towards our INDIA and its ancient culture.
- Now I wish to learn Sanskrit to know more about our ancient Scientific developments.
- Every person on earth should have the knowledge of Vedic Mathematics.
- It was a boon to learn Vedic Mathematics in Dushera holidays (I had planned for my regular studies).
- Want to learn more about Vedic Mathematics like factorisation etc, which are difficult in modern system.
- Amazing speed, very wide applications covering almost all the mathematical operations.
- A dynamic step for its introduction in educational system is a must.
- Has created an interest to know more about it.
- It can be of great use for children if made part of sysyllabus.

Contd.......2/-

- Mysterious nature of numbers. After little practice problems can b mentally solved at magic speed.
- Very educative, has much scope for coming generation to start its practice from beginning.
- It is a strong proof of our rich cultural heritage.
- By Vedic methods maths which is a burdoh, be-comes a joy for the children.

This is the most interesting and thrilling time of my life. Being a mathematics teacher I have interest in maths but this Vedic Mathematics has thrilled me. When I came in contact with the Vedic Mathematics it gave such a pleasure which I can't explain in words.

A teacher of Adarsh Bal Niketan

- The lectureswere eye openers and most thrilling
- A glimpse of the lofty trakking in Himalayas of ancient knowledge i Maths, developed with spiritualism. Cudos to SSG.

A Professor of University

THE TIMES OF INDIA, NEW DELHI FRIDAY, NOVEMBER 27, 1987

Project to study use of Vedic Maths in Computers

New Delhi, November 26 (UNI) The recently set-up Rashtriya Ve Vidya Pratishthan (National foundation for Vedic studies) has been asked to prepare a project to examine the applicability of ancient Vedic Mathematics to modern computer calculations, the human resource development minister. Mr. P.V. Narasimha Rao, told the Lok Sabha today.

Replying to supplementaries by Dr. A. K. Patel, he said this was decided after researchers at the California university found out that Indian mathematicians of the Vedic period knew and successfully employed the binary and other numbers which formed the basis of the modern computer concept.

This news has also appeared in

1. PATRIKA, Kanpur Friday Nov.27, 1987 issue Ancient Capability in hi-tech calculations.
2. Aaj - Kanpur Nov. 27, 1987 (Hindi)
3. Navjeevan Lucknow Nov. 27, 1987 (Hindi)
4. Nav bharat New Delhi, Nov. 27, 1987 (Hindi)
5. National Herald Nov. 27, 1987 (English)
6. Hindustan Delhi Nov. 27, 1987 (Hindi)
7. Detailed report was Published in The Hindustan Times, Nov, 27 1987 (F).

आध्यात्मिक अध्ययन संस्था (आडस)
वैदिक विज्ञान संस्थान

SPIRITUAL STUDY GROUP (SSG)
Institute of Vedic Sciences
41/2, Amod Kunj
University Campus
ROORKEE-247 667 (INDIA)

WORKSHOP on VEDIC MATHEMATICS
and its
COMPUTATIONAL POTENTIALITIES
(Advanced)

वैदिक गणित अध्ययन

NAME	DEPARTMENT
Indra Gupta	E.E.S. Research Fellow
Uma Arora	Mathematics, S.R.F.
Abhishek Srivastava	B.E. III Civil
Miss Vandana Sudhera	St. Ann's School, Teacher
F.S. Umrigar	Civil (R/S)
Nimish Minal,	Adarsh Bal Niketen XI
Anita Rampal	Teacher St Ann's
Jaydev J. Bhavsar	Civl Engg. Lect.
Prabhaker Reddey	
Briti Dass	XI
Smt. Renu Bhargava	Warden S' Flat, Lec. CED
Raj Kumar Verma	K.L.D.A.V. RKE, Teacher

N.B. In addition to the participants listed under the elementary course these participants join only the advance sessions of the workshop.

Delegates who attended the Vedic Maths Workshop without formal registration

Dr. D. R. Khanduja	Prof. in Mech. Engg. UOR
Sri. Dila Ram	School Teacher Saharanpur
Sh. S. yadav	,, ,, ,, Roorkee
Sri. S. Groover	Reader in UOR
Sri. R.K. Kakati	Senior Research Fellow
Ramesh Choudhary	School Teacher
Sri. D.P. Sharma	Senior Research Fellow
Sri. R.S. Nayak	Senior Research Fellow
Sri Vikas Sehgal	School Teacher
Dr. A.M.C. Srivastava	Reader in Civil Engg.
Sri. W.S. Badarinath	Reader in W.R.D.T.C.
Prof. G.P. Srivastava	Prof. in W.R.D.T.C.
Dr. M.K. Mittal	Prof. in Mech. UOR
Dr. S.N. Bandey	Prof. in Maths. UOR
Dr. U.S. Gupta	Reader in Maths UOR
Dr. M.L. Gaindhar	Prof. in Mechanical Engg. Deptt.
Dr. G.S. Aggarwal	Reader in Maths UOR
Dr. H.G. Sharma	Reader in Mech. UOR
Dr. T.S. Raju	Lecturer Maths S. India, Rohtak
Dr. S.K. Kapoor	Senior Judicial Megistrate
Dr. Jagdish Parshad	Ledturer in civil Engg. deptt

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
Vedas are the Eternal Sources of All Knowledge

PANEL DISCUSSION 1
ON
PERSPECTIVE PLANNING OF VEDIC MATHEMATICS

held on 16th August 1987
(Sri Krishen Janamastmi)
in the Conference room of Civil Engineering Department
University of Roorkee, Roorkee.

Chairman	Prof. A.S. Arya Prof. & Head Earthquake Engineering Deptt. University of Roorkee
Chief Guest	Prof. N. C. Mathur Vice-Chancellor University of Roorkee
Key Note speaker	Prof. A. Nicholas - LONDON
Organising Secretary	Dr. N. Puri
Special invitees.	
Mr. C. P. Nigam	Director, Rajya Vigyan Sansthan ALLAHABAD.
Dr. S. R. Agarwal	Department of P Medicine, Lady Harding College New Delhi
Dr. G. M. Verma	Secretary, Vidya Bharati New Delhi
Shri. S. K. Kapoor	Senior Judicial Majistrate ROHTAK
Shri. S. K. Taneja	Executive Engineer DDA DELHI
Shri. [illegible]. N. Agarwal	Yoga Teacher Chandigarh
Swami Keshwanandaji	Divine life Society Hrishikesh
Mr. M. N. Tiwari	P & T Kanpur
Dr. G.C. Nayak	Prof. & Head, Civil Engg. Deptt. U. O. R. Roorkee.
Dr. S.N. Pandey	Prof. Maths. Deptt. U.O.R.
Dr. G. S. Agarwal	Reader, Maths. Deptt. U.O.R.
Dr. J. L. Gaindhar	Prof. Mechanical Engg. Deptt. U.O.R.
Dr. J.P. Singhal	Reader, Mechanical Engg. Deptt.U.O.R.
Prof. G. P. Srivastava	Prof. W. R. D. T. C.
Sh. H. S. Badrinath	Reader, W. R. D. T. C.
Dr. P.S. Mishra	Reader, Metallurgical Engg. Dept.U.O.R.
Sh. T.N.S. Mathur	Reader, Chemical Engg. Deptt. U.O.R.
Dr. P. D. Porey	Lecturer, Civil Engg. Deptt. U.O.R.
Sh. Himanshu Joshi	Research Scientist, Civil Engg.
Miss Anita Rampal	Teacher St Ann's School
Mrs. Usha Rani	Teacher Adarsh Bal Niketan
Miss Pragati Goel	SRF Maths. Deptt.
Sh. Sandeep Nayyar	CAD, Centre Civil Engg.
Miss Archana Tiwari	M.E.(Final) Civil Engg.
Miss Indra Gupta	Research Associate, Electrical Engg.
Sh. Anurag	M.E. (Final) Electronics Engg.
Miss Shruti Mishra	XII Class Adarsh Bal Niketan
Miss Parul Mathur	XII class St. Ann's School

Representative of All India Radio Nazibabad.

आध्यात्मिक अध्ययन संस्था (आऽस)

SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE-247 667 (INDIA)

वैदिक गणित अध्ययन

WORKSHOP PROGRAMME

This workshop on Vedic Mathematics consisted of several expert lectures alongwith practice sessions. The brief outline of different sessions is as under :

DATE	Topic	Teachers
7. 8. 87	Inaugration and Inaugral add.	Dr. A.S. Arya , Head, Eq. Engg.
	Keynote lecture	Dr. N. Puri
	Nikhilam - Multiplication	Dr. N. Puri
8. 8. 87	Urdhava, Recurring decimals ' Vedic Code & Vinculum	Prof. A. Nicholas Dr. N. Puri
	Practice Session	Dr. P.S. Mishra Km. Pragati Goel
9. 8. 87	Lecture - Nikhilam division Paravartaya division and checking of division	Dr. P.S. Mishra
	Practice Session	Shri H.S. Badrinath Dr. P.S. Mishra
10. 8. 87	Maths on fingures	Shri T.R. Raju
	General method of division and Algebraic division	Prof. A. Nicholas
	Practice Session	Dr. P.S. Mishra Km. Pragati Goel
11. 8. 87	Maths on fingers -II	Shri T. R. Raju
	Algebra & Factorisation	Prof. A. Nicholas
	Practice Session	Dr. P.S. Mishra Shri H. S. Badrinath
12. 8. 87	Integration in one line including $e^{-x^2/2}$	Prof. A. Nicholas
	Practice Session	Dr. N. Puri Km. Pragati Goel
	Evaluation of determinants and inversion of Matrices	Prof. A. Nicholas
13. 8. 87	Solution of Simultaneous Equation	Prof. A. Nicholas
	Practice Session	Dr. P.S. Mishra Dr. N. Puri.
	Quadratic and Cubic equation	Prof. A. Nicholas

वेदात् सर्वं प्रसिध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
Vedas are the Eternal Sources of All Knowledge

14. 8. 87	An Overview of Vedic Maths, nonlinear differential equations, ordinary and partial	Prof. A. Nicholas
	Practice Session	Dr. G.S. Agarwal Km. Pragati Goel
	Trigonometrical functions	Prof. A. Nicholas
15. 8. 87	Mathematical basis of Vedic Texts	Mr. S.K. Kapoor
	Practice Session	Dr. N. Puri Dr. P.S. Mishra
	Kepler's Equation	Prof. A. Nicholas
16. 8. 87	Alpha Numeric Code	Dr. S.N. Agarwal
	Calculus	Prof. A. Nicholas
17. 8. 87	Differential Equation Linear and nonlinear	Prof. A. Nicholas

OPINIONS OF PARTICIPANTS

- There is an urgent need that there things should get exposed to the messes and get it pupularized.

- The programme was well designed the Sepeakers were experts & well chosen for their dedication to Vedic Mathematics and exponents like Shankaracharya. The lecture were eye opener an most thrilling. A glimpse of the lofty trakking in Himalyays o ancient knowledge in Maths, developed with Spiritualism. Cudos to SSG.

The panel discussion of expert educationists planned on 16th Aug., 1987 .

Theme :

" Perspective Planning of Vedic Mathematics and its Computational Potentialities. "

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आsस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

वैदिक गणित अध्ययन

THE TIMES OF INDIA, NEW DELHI
FRIDAY, NOVEMBER 27, 1987

Project to study
use of Vedic
maths in computers

NEW DELHI, November 26 (UNI) The recently set-up Rashtriya Ved Vidya Pratishthan (National foundation for Vedic studies) has been asked to prepare a project to examine the applicability of ancient Vedic mathematics to modern computer calculations, the humnan resource development minister. Mr. P. V. Narasimha Rao, told the Lok Sabha todya.

Replying to supplementaries by Dr. A. K. Patel, he said this was decided after researchers at the California university found out that Indian mathematicians of the Vedic period knew and successfully employed the binary and other numbers which formed the basis of the modern computer concept.

Mr. Rao said the pratishthan, primarily set up " three-four months back " would try to relate the Vedas, the Vedangas and Upvedas to modern times.

The human resource development minister said the great temples of Indian, which have " stood the test of time" demonstrat the precision of Vedic mathematics employed in their complicated design and architecture.

This news has also appeared in

1. PATRIKA, Kanpur Friday Nov. 27,1987 issue Ancient Capabilit in hi-tech calculations.
2. Aaj - Kanpur Nov. 27,1987(Hindi)
3. Navjeevan Lucknow Nov. 27,1987, (Hindi)
4. Nav bharat New Delhi, Nov. 27,1987 (Hindi)
5. National Herald Nov. 27,1987 (English)
6. Hindustan Delhi Nov. 27,1987 (Hindi)
7. Detailed report in Hindustan is reproduced at the back etc. etc.

P.T.O.

वेदात् सर्वं प्रसिद्धयति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

THE HINDUSTAN TIMES

New Delhi Friday November 27, 1987

VEDIC INDIANS KNEW OF BINARY SYSTEM

Hindustan Times Correspondent

New Delhi , Nov. 26 - The binary system (zero and one) which is the base of the entire computer(s) system was available in the Veda thousands of years ago. Minister for Human Resource Development P.V. Narasimha Rao said in the Lok Sabha today.

Replying to a question on Vedic mathematics by BJP member A. K. Patel, Mr. Rao said that a well-known Vedic scholar, the Shankaracharya of the Govardhan Peeth, had brought our a volume in the sixties in which he had given 16 " sutras " in Vedic lore an proved that all complicated computations could be solved on the basis of these sutras more easily than was being followed so far.

Mr. Rao, himself a reputed scholar, said the new mathematics introduced in the country in the sixties to which objections were raised was based on numbers. Today's computers too were based on numbers, Apart from the decimal system, (which had 10 as the base) there was a base of 16 as well as two (zero and one) now used in the computer system. These were available in the Vedas.

He told Dr. Patel that researchers in Britain had found that the Vedic mathematics was useful in checking results of comput calculations and was opening new vistas in other fields too. They were amazed, Mr. Rao said, that apart from the gift of zero to the world of mathematics, ancient Indians, were familar with the binary system too.

Some scholars in India, he said in reply to the main question tabled by the BJP member, were known to be engaged in the study and propagation of Vedic mathematics. Rashtriya Ved Pratistha which had recently been set up, was considering the preparation of project to encourage Vedic mathematics and its application to computer calculations.

The Pratisthan was set up only three or four months ag but the results seemed to be encouraging, the Minister said.

Mr. Rao said these lores were available in what were known as the Upa-Vedas. For example "sthapatya"(architecture), was known in ancient India and the temples built on the basis of such knowledge had stood the test of time.

"We are all interested to see what we can give back to the west which we know thousands of years ago" he said.

आध्यात्मिक अध्ययन संस्था (आअस)

SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE-247 667 (INDIA)

वैदिक गणित अध्ययन

WORKSHOP ON VEDIC MATHS

ELEMENTARY COURSE

VEDIC MATHS

A. PUSHP :- 1

S.No.	Chapter No.	Title
1	1	Base System
2	3	Vinculum
3.	10	Multiplication
4	13	The General Method of Multiplication
5	14	Multiplication check
6.	15	Recurring decimals

B. PUSHP - II

7.	29	Nikhilam division
8.	30	Paravartaya division
9.	31	Algebraic division
10.	32	Straight division
11.	33	Division check
12.	38	Vedic Numeric Code.

Note: Advance Course is in the later half.

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं।
Vedas are the Eternal Sources of All Knowledge

CHAPTER-1

BASE SYSTEM

In Vedic Mathematics the base system is very frequently used to reduce the computations involved. The base is considered to be the powers of 10. For example 10, 100, 1000, 10,000 and so on are the different bases.

Deviation from the Base:

1. We should take, as base, for our calculations, that power of 10 which is nearest to the number.
2. For the numbers more than the base, the portion of the number exceeding the base is written with a +ve sign in front of the number.
3. For the number below the base, we substract the number from the base and write down the remainder in front of the number with a -ve sign. This deviation is directly written by using the Nikhilam Sutra. (Sutra No.2). The Sutra means All from 9 and last from 10'.

Examples:

1.1 Say Number is 103
The base is 100 and the number is 3 more than the base.
Therefore, 103 is written as 103+03

1.2 Say Number is 115
The base is 100 and the number is 15 more than the base.
Therefore, 115 is written as: 115+15

1.3 Say Number is 1012
Now base is 1000 and the number is 12 more than the base.
Therefore, 1012 is written as: 1012+012

1.4 Say Number is 13
This time base is 10 and the number is 3 more than the base
Therefore, 13 is written as: 13+3

1.5 Say Number is 10004
This time base is...... and the number is ....
more than the base.
Therefore .... is written as ...: ...+ .....

1.6 Say the Number is 9
The base is 10 and 10 - 9 gives a remainder of 1,
therefore 9 is one below the base 10.
The number 9 is written as: 9 - 1

1.7 Say the number is 92
The base is 100 and the number is 8 less than the base
Therefore 92 is written as: 92 - 08

1.8 Say the number is 991.
The base is 1000 and the number is 9 below than the base.
Therefore, 991 is written as: 991 -009
By Nikhilam Sutra we get 9-9=0.9-9=0and 10-1=9
Therefore deviation = - 009.

1.9 Say the Number is 75.
The base is 100 and the number is 25 less than the base.
Therefore,. 75 is written as: 72-25

1.10 Say the Number is 971
The base is 1000 and the Number is 29 less than the base
Therefore, 971 is written as: 971-029
By Nikhilam Sutra we get 9-9=0; 9 - 7 = 2; and 10-1=9
Therefore deviation = -029.

1.11 Say the number is 9975.
The base is ..... and the number is .... less than the base.
Therefore .... is written as: ....

Note: If the difference of the number from the base is very large, a suitable sub-base (or working base) can also be used. The sub-base is a convenient multiple of a suitable base. For example the number is 41. If we take a base of 10, it is 31 more than 10. If we take a base of 100, it is 59 less than 100. Therefore, it shall not be convenient to use a base of 10 or 100 and a working base shall have to be used. (To be explained later on, in subsequent chapters.).

CHAPTER 3

Vinculum

3.1 INTRODUCTION

In the previous chapter we have seen how we can use the Nikhilam Sutra and reduce any of the operators for writing tables to be at the most equal to 5. This is a unique facet of Vedic Mathematics. Through this process the computations become far more easier than otherwise. As a matter of fact the unique concept of Vinculum in the Vedic mathematics is a very interesting and useful device.

We are generally not familiar with this approach, therefore before using this Vinculum approach for addition, multiplication, division and squaring etc. it is important to become familiar with it. Through this approach we can make it sure that we have almost never to operate digits larger than 5.

3.2 THE APPROACH

In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to the next higher order digit.

Illustration 1 : Let us take the number 9. 9=10-1, therefore 9 is 1 less than 10. This we can also write as $1\bar{1}$, where $\bar{1}$ at the unit place means 1 with a negative sign. For further operations we can use $1\bar{1}$ instead of 9. The property of the numbers is unchanged and we can add, multiply etc. in the usual manner. However, we have to keep in mind that the $\bar{1}$ is always appearing as -1.

In the reverse order, when we need to normalise,

$1\bar{1} = 10 - 1 = 9$

Illustration 2 : 29 = 30 - 1.

This can be written as $3\bar{1}$.

Illustration 3 : $37 = 40 - 3 = 4\bar{3}$

So we find that we take the complement of the digit larger than 5 and write this with a bar indicating the negative sign, and increase the digit at the next higher place.

Illustration 4 : 79 = 80 - 1 = $8\bar{1}$

Further 80 = 100 - 20 = $1\bar{2}\bar{0}$

therefore 79 = $8\bar{1}$ = $1\bar{2}\bar{1}$

So we find 79 can be written as $8\bar{1}$ but since the digit a the tenth place is also very big we can take its complement (from 10, 10-8 = 2) and increase the digit previous to this by (0+1 = 1, therefore 79 becomes $1\bar{2}\bar{1}$)

It is interesting to note that if there are a few digits larger than 5 appearing together, their collective complement can be directly written by using the Nikhilam Sutra and increas the previous digit by 1. In this case 9-7 = 2, 10-9 = 1, giving the complement as 21 and the previous digit 0 becomes 0 + 1 = 1 Hence 79 = $1\bar{2}\bar{1}$.

Illustration 5 : 3789

By Nikhilam Sutra, the complement of the last three digi is 9-7 = 2, 9 - 8 = 1, 10-9 = 1, and the previous digit gets increased by 1 (3+1 = 4). Therefore,

3789 = $4\bar{2}\bar{1}\bar{1}$

Illustration 6 : 286923

The complement of the larger digits (869) by Nikhilam Su is 131. Therefore

286923 = $3\bar{1}\bar{3}\bar{1}23$

Note that the last digits 2 and 3 at the tenth and the unit pl stay on as before, and the digit previous to the bigger ones ge increased by 1.

Illustration 7 : 353782

353782 = $354\bar{2}\bar{2}2$

(The complement of 78 is 22)

Illustration 8 : 3728193

$3728193 = 4\bar{3}3\bar{2}2\bar{1}3$

The application of the Vinculum process should be well practised before we use it. However, its use makes the computation process quite easy and fast.

Illustration 9 : 782893

$782893 = 1\bar{2}\bar{2}3\bar{1}\bar{1}3$

Let us further consider a few examples of normalizing the Vinculum number. It shall be suprisingly interesting to learn that the Nikhilam sutra can be used even for the normalizing process. The only difference being that the previous digit is now reduced by 1.

Illustration 10 : $3\bar{3}4\bar{4}$

The complement of $\bar{3}$ is 7 and of $\bar{4}$ is 6, Further, each digit before the bar digit gets reduced by 1. So that 3-1 = 2 and 4-1 = 3.

Therefore $3\bar{3}4\bar{4} = 2736$

Illustration 11 : $3\bar{3}\bar{4}\bar{2}$

Since there are three digits in a group we can use the Nikhilam sutra to straight away write down the complement of this group, and we should remember to reduce the digit previous to this group by 1.

Therefore, by Nikhilam 9 - 3 = 6, 9 - 4 = 5 and 10-2 = 8, and 3 - 1 = 2.

As such $3\bar{3}\bar{4}\bar{2} = 2658$

Illustration 12 : $2\bar{3}\bar{2}6\bar{1}\bar{1}2$

Therefore,

$2\bar{3}\bar{2}6\bar{1}\bar{1}2 = 1685892$

Illustration 13 : $20\bar{3}\bar{4}0\bar{1}\bar{2}1$

$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 19659881$

Alternatively, we can normalize the above number in two steps

$20\bar{3}\bar{4}0\bar{1}\bar{2}1 = 2\bar{1}66\bar{1}881$

$= 19659881$

Illustration 14 : $3\bar{6}2\bar{2}\bar{7}$

$3\bar{6}2\bar{2}\bar{7} = 24173$

Note:- We may not feel very comfortable to see the numbers bigger than 5 appearing with a bar, i.e. in the Vinculum form, as we would not be generally converting the smaller digits into their Vinculum. However, during the computations of the Vinculum numbers we may frequently end up in a situation like this.

Discussion :- We shall operate with the vinculum numbers exactly in the same manner as we do with the ordinary numbers without vinculum. In other words, we can understand that the property of these numbers is unchanged. However, we handle them alongwith their minus sign in the usual manner.

Illustration 15 : $40\bar{2}\bar{1}\bar{3}1$

Therefore, $40\bar{2}\bar{1}\bar{3}1 = 397871$

In two steps we can say

$40\bar{2}\bar{1}\bar{3}1 = 4\bar{1}7871$

$= 397871$

Further practise shall give us the grip on their system, which is immensely useful for simplifying the computation processes.

CHAPTER 7

COMPOUND SUBTRACTION

7.1 Compound Subtraction:

The process of compound subtraction in Vedic Method is converted to a series of simple substraction and addition. Once again the Sutra Shudha is deployed whenever the larger number of a particular unit is to be substracted from the smaller number of the same unit. It is to be noted that in Vedic Method there is absolutely no need to convert different units to a single unit as is frequently done in the normal procedure. The step by step process is illustrated in the following examples:

7.2.1 Example 1:

$$\begin{array}{rrr} 5 \text{ yds} & 1 \text{ ft} & 9 \text{ inches} \\ -\ 2 & 2 & 11 \\ \hline \\ \hline \end{array}$$

Step 1:

The number 11 at inches place is bigger than the corresponding no.9. Therefore Shudhikaran is required.

(a) We say Shudha and put a dot above the lower number at the next unit (2).

(b) Find the complement of the lower number from the unit size (12) in this case as there are + 12 inches in 1 ft. Therefore 12 - 11 = 1 is the complement of the lower number 11 (from 1 ft the next unit in this case).

(c) Add this complement to the upper number. 1 + 9 = 10 is the answer.

$$\begin{array}{rrr} 5 \text{ yd} & 1 \text{ ft} & 9 \text{ inch} \\ -\ 2 & \dot{2} & 11 \\ \hline & & 10 \\ \hline \end{array}$$

Step 2:

We move on to next unit (which is ft in this case). Since the lower number has a dot above this, we increase it by

1. Therefore 2 + 1 = 3 and 3 is larger than the corresponding number at this place (which is 1 in this case). Sudhikaran is required.

(a) We say Sudha and put a dot above the lower number at the next unit (2 yds here).

(b) The complément of this lower number is computed by subtracting it from the unit size, which is 3 in this case (as there are 3 ft in 1 yard). Therefore 3 - 3 = 0 is the complément of the increased lower number 3.

(c) Add this complément to the upper number. Therefore 1 + 0=1 is the answer.

$$\begin{array}{rrr} 5 \text{ yd} & 1 \text{ ft} & 9 \text{ inches} \\ -\dot{2} & \dot{2} & 11 \\ \hline & 1 & 10 \\ \hline \end{array}$$

Step 3:

We move on to the next unit which is yards in this case, Since the lower number has a dot above this, we increase it by 1.

Therefore 2 + 1 = 3. In this case the lower number (3) is smaller than the upper number (5) as such 5 - 3 = 2 yards is the answer. Therefore the final answer is 2 yards 1 ft and 10 inches.

NOTE: We observe that in the Vedic Method of compound subtraction there is no carry over of the unit instead we only find out the complement of each number. The later being easier, hastens the computation process and also reduces the chances of an error.

7.2.2 Illustration 2:-

$$\begin{array}{lllllll} 1 \text{ Decade} & 3 \text{ yrs} & 3 \text{ months} & 5 \text{ days} & 13 \text{ hrs} & 50 \text{ min} & 30 \text{ seconds} \\ 0 & 2 & 4 & 5 & 15 & 55 & 25 \\ \hline \\ \hline \end{array}$$

Step 1:

30 - 25 = 5 secs is the straight answer for the first unit (secs in this case).

Step 2:

55 min is larger than the corresponding upper number (50). Therefore Shudhikaran is carried on. We put a dot above the number at the next unit (15 hrs). The complement of 55 is 5 (60-55), which is added to the upper number (50), therefore 5 + 50 = 55.

1 decade	3 yrs	3 months	5 days	13 hrs	50 min	30 seconds
0	2	4	5	$\dot{15}$	55	25
					55	5

Step 3:

Move to the next unit (hrs). We find a dot above the number which therefore gets increased by 1 (15+1=16). The lower number is once again larger than the upper number (13) therefore the Shudhikaran process is repeated. We put a dot above the next number (5). Further the complement of 16 is 8(24-16) which is added to the upper number 13, therefore 8 + 13 = 21 is the answer.

1 decade	3 yrs	3 months	5 days	13 hrs	50 min	30 secs.
0	2	4	$\dot{5}$	$\dot{15}$	55	25
				21	55	5

Step 4:

Move to the next unit (days) we find a dot above the number therefore 5 + 1 = 6. The lower number being larger, Shudhikaran is performed. Put a dot at the top of the next number (4). Compute the compliment of 6(30 - 6 = 24). Add 24 to the upper number. Therefore the answer is 24 + 5 = 29.

Step 5:

Similarly the process is repeated to complete the computation.

1 decade	3 yrs	3 months	5 days	13 hrs	50 min	30 secs
0	$\dot{2}$	$\dot{4}$	$\dot{5}$	$\dot{15}$	55	25
1	0	10	29	21	55	5

7.2.3 ILLUSTRATION 3

$$\begin{array}{rlll} & 2 \text{ hrs} & 31 \text{ min} & 29 \text{ sec} \\ - & 0 & 5\dot{9} & 42 \\ \hline & 1 \text{ hrs} & 31 \text{ min} & 47 \text{ sec} \end{array}$$

Step 1: Finding conversions

60 sec = 1 min; 60 min = 1 hr

Step 2: Find the deviation of the bigger no. from 60 i.e. 60 sec - 42 sec = 18 sec. Now we add 18 to 29 sec., we get 47 sec.

Step 3: Similarily 59 + 1 = 60

60 -60 = 0

than 0 +31 = 31 min

Step 4: 0 + 1 hr = 1 hr

than 2 - 1 = 1 hr

Hence the result is 1 hr 31 min 11 sec

CHAPTER 10

MULTIPLICATION

10.1 Introduction:

It is interesting to note that there is a general formula which can be applied to all cases of multiplication. This formula is quite easy and simple. However, there are special cases where special Vedic procedure gives much simpler, easier and quicker solutions. The latter shall be discussed first.

10.2 Nikhilam Sutra:

It reads Nikhilam Navatascaramam Dasatah . The literal translation of this crisp Sutra is

All from 9 and the last from 10.

The multiplication procedure based on this Sutra is known as the Nikhilam method. This is very convenient method for multiplying numbers which are close to the base (refer chapter 1). The exact use of this Sutra shall be elaborated in later sections.

Now the step by step procedure is outlined in the following sections.

10.3 TYPE-1

10.3.1 Let us take the simple example of multiplying 13 by 11.

Step 1: We write these numbers in the Vedic base system style and and get: (Refer chapter 1)

13 + 3
11 + 1

We note that the base is 10 and there is only 1 zero in the base. Positive sign indicates that both numbers are greater than the base.

Step 2: The product will have two parts. An inclined dividing line should be drawn to demarcate these two parts

13 + 3
11 + 1

Step 3: We vertically multiply the excess digits from base.

This product gives us the right hand side portion of the answer. In this case 3 x 1 = 3 is the product.

$$\begin{array}{r} 13 + 3 \\ \underline{11 + 1} \\ /\ 3 \end{array}$$

Step 4:(a) The left hand side of the answer can be arrived at by simply adding diagonally any of the two given numbers with the deviation (deficit) of the other number from base.

Case 1: The first number 13 is added to 1, the deficit of second number. Therefore 13 + 1 = 14

Case 2: The second number 11 is added to 3 the deficit of the first number. Therefore 11 + 3 = 14

Note: Both of these alternatives gives the same result

$$\begin{array}{r} 13 + 3 \\ \underline{11 + 1} \\ 14/\ \ 3 \end{array}$$

Step (b): In addition to the above two alternatives the left hand side can also be obtained by following two additional procedures.

Case 1: Subtract the base from the sum of the given numbers. Here the base is 10 and the sum of the given numbers is 13 + 11 = 24. Therefore, sum-base = 24-10 = 14.

Case 2: Add the sum of the two deficits to the base. Here the sum of the two deficits is +3 + 1 = +4.
Therefore, base + sum = 10+4 = 14

Both of these alternatives also give the same correct L.H.S. of the answer.

Step 5: Thus 11 x 13 = 143

$$\begin{array}{r} 13 + 3 \\ 11 + 1 \\ \hline 14\ /\ 3 \end{array}$$

Note: The Vedic System has this special feature of availability of the same result in several easy ways. This feature can be utilised by the student to verify his answer step by step.

10.3.2 Further Illustrations

14 + 4	11 + 1	19 + 9	12 + 2	17 + 7	13 + 3
12 + 2	15 + 5	11 + 1	13 + 3	11 + 1	13 + 3
16 / 8	16 / 5	20 / 9	15 / 6	18 / 7	16 / 9

An Important point to be observed is that in each of these cases the multiplication of the deficit digits yields a single digit. Since the bases in all of them is 10, it has only one zero, and as such the single digit answer on the right hand side creates no problem.

10.4 TYPE-2:

In case of the multiplication problems where the deficit multiplication yields digits more than the number of zeros in the base, the excess digits are carried over to the left hand side and added to left hand answer obtained in the above mentioned way (Step 4).

10.4.1 ILLUSTRATIONS: (i) 14 x 13

$$\begin{array}{r} 14 + 4 \\ \underline{13 + 3} \\ 17\ /_{1}2 \end{array} = 17 + 1/2 = 18/2 = 182$$

(ii) 16 x 12

$$\begin{array}{r} 16 + 6 \\ \underline{12 + 2} \\ 18\ /_{1}2 \end{array} = 18 + 1/2 = 19/2 = 192$$

(iii) 18 x 13

$$\begin{array}{r} 18 + 8 \\ \underline{13 + 3} \\ 21\ /_{2}4 \end{array} = 21 + 2/4 = 23/4 = 234$$

Let us take some illustrations with higher bases:

(iv) 106 x 105

$$\begin{array}{r} 106 + 6 \\ \underline{105 + 5} \\ 111\ /30 \end{array}$$

In this illustration the product of the deficit numbers is 6 x 5 = 30. And it has only two digits which are equal to number of zeros in the base (base is 100!). Therefore, two digits are permissible on the r.h.s. and no carry over is required.

(v)
109 + 9 | 108 + 8 | 102 + 2
111 + 12 | 109 + 9 | 144 + 44
120 /99 | 117 /72 | 146 / 88

(vi) 111 x 112

111 + 11

112 + 12

$123\ /_{1}32$ = 123 + 1/32 = 124/32 = 12432

Note: The r.h.s. should be obtained by mentally performing the vedic method i.e.

11 + 1

12 + 2

13 / 2

(vii) 113 x 114

113 + 13
114 + 14
$127\ /_{1}82$ = 127 + 1/82 = 128/82
= 12882

Solving R.H.S.

13 + 3
14 + 4
18 / 2

(viii) 117 x 118

117 + 17
118 + 18
$135\ /_{3}06$ = 135+3/06 = 13806

Solving R.H.S.

17 + 7
18 + 8
30 / 6

10.5 TYPE 3:

The previous two types have illustrated the point that the number of digits on the right hand side of the answer must be equal to the number of zeros in the base. In case the number of digits exceed the number of zeros in the base, the excess digit is carried over to the L.H.S. However, if the multiplication of the deficit digits yields the R.H.S. having digits less than the number of zeros in the base, then we prefix the zeros to the R.H.S. so that the total number of digits is equal to the number of zeros in the base.

10.5.1 Multiply 102 x 103

102 + 2
103 + 3
105 /06

The R.H.S. = 3 x 2 = 6. Since the base 100 has two zeros.

therefore, we prefix one zero before 6 i.e. we write 06 as the R.H.S.

10.5.2 ILLUSTRATIONS

$$\begin{array}{r} 107 \times 101 \\ 107 + 7 \\ 101 + 1 \\ \hline 108\,/07 \end{array} \qquad \begin{array}{r} 104 \times 102 \\ 104 + 4 \\ 102 + 2 \\ \hline 106\,/08 \end{array} \qquad \begin{array}{r} 1010 \times 1005 \\ 1010 + 10 \\ 1005 + 05 \\ \hline 1015\,/050 \end{array}$$

$$\begin{array}{r} 10010 \times 10101 \\ 10010 + 10 \\ 10101 + 101 \\ \hline 10111/\,1010 \end{array} \qquad \begin{array}{r} 1099 \times 1001 \\ 1099 + 99 \\ 1001 + 01 \\ \hline 1100/\,099 \end{array}$$

103×2312

The base as 100

$$\begin{array}{l} 103 + 3 \\ 2312 + 2212 \\ \hline 2315\,/_{66}36 = 2315 + 66/36 = 2381/36 = 238136 \end{array}$$

103×13

Taking base as 10

$$\begin{array}{l} 103 + 93 \\ \;\;13 + 3 \\ \hline 106\,/_{27}9 = 106+27/9 = 133/9 = 1339 \end{array}$$

10.6 TYPE 4:

In the three types of problems discussed at least one of the numbers to be multiplied was just a little more than the certain power of 10 which was taken as a base. However, if the numbers to be multiplied are just a little below the base, the steps to be followed are just the same. (As already explained in previous cases) The only difference is that instead of cross addition for obtaining the L.H.S. of the answer we do the cross subtraction. However, the other rules regarding digit surplus and digit deficit are exactly the same.

10.6.1 ILLUSTRATIONS

$$\begin{array}{c} 9 \times 6 \\ 9 - 1 \\ 6 - 4 \\ \hline 5\ /\ 4 \end{array} \qquad \begin{array}{c} 6 \times 5 \\ 6 - 4 \\ 5 - 5 \\ \hline 1\ /_{2}0 \end{array} = 1+2/0=3/0 = 30$$

8 x 6

8 - 2
6 - 4
4 / 8

91 x 93

91 - 9
93 - 7
84 / 63

89 x 97

89 - 11
97 - 3
86 / 33

64 x 98

64 - 36
98 - 2
62 / 72

79 x 97

79 - 21
97 - 3
76 / 63

988 x 998

988 - 12
998 - 2
986 /024

985 x 995

985 - 15
995 - 5
980 - 075

99976 x 99998

99976 - 24
99998 - 2
99974/ 00048

Note:- Please note that the deviation of these numbers which are less than the base is obtained directly by using the NIKHILAM SUTRA. For example, the deviation of the number 99976 from the base of 100,000 is obtained by Nikhilam Sutra, All from nine and last from ten . Therefore, 9-9 = 0; 9 - 9 = 0; 9 - 9 = 0; 9 - 7 = 2; and 10 - 6 = 4. Hence our deviation is - 00024 etc.

10.7 TYPE 5:

If one of the multiplying numbers is more than the base and the other is less than the base, then the deficits shall be positive and negative respectively. The right hand side shall therefore, be negative and the same shall have to be subtracted from the L.H.S. for obtaining the answer.

10.7.1 ILLUSTRATIONS

13 + 3
9 - 1
12 / $\bar{3}$ = 11/10-3 = 11/7 = 117

106 + 6
96 - 4
102 /$\overline{24}$ = 101/100-24 = 101/76 = 10176

108 + 8
91 - 9
99 / $\overline{72}$ = 98/100-72 = 9828

$$\begin{array}{l} 103 + 3 \\ \underline{97 - 3} \\ 100/\,\overline{09} \end{array} = 99/100-09 = 9991$$

$$\begin{array}{l} 1021 + 21 \\ \underline{996 - 4} \\ 1017\,/0\overline{84} \end{array} = 1016/1000-84 = 1016916$$

$$\begin{array}{l} 10012 + 12 \\ \underline{9997 - 3} \\ 10009/\,00\overline{36} \end{array} = 10008/\ 10000-0036 = 100089964$$

Note: The right hand portion of the answer is with a -ve sign. It is called the Vinculum-portion'. Note that even the subtraction of the Vinculum portion can be easily done with the aid of the Sutra Nikhilam Navatascaramam Dasatah (all from nine and the last from 10).

CHAPTER 12

NIKHILAM COROLLRIES

12.1 INTRODUCTION: The previously illustrated examples were all solved by using the Nikhilam Sutra. Some of these examples can be more easily done by using Urdhva-Tiryagbhyam Sutra. This sutra is the general formula applicable for all types of multiplication.

However, there are a few more corollaries arising out of the Nikhilam Sutra discussed in the previous chapter. He shall first learn these corollaries before discussing the Urdhva Tiryagbhyam sutra.

12.2 THE FIRST COROLLARY

The first corollary of Nikhilam Sutra is Yavadunam Tavadunikrtya Varganca Yojayet . It means what ever the extent of its deficiency lessen it further to that very extent; and also set up the square (of that deficiency).

This sutra essentially deals with squaring of the numbers.

12.2.1 Illustrations

CASE 1: The numbers smaller than the base i.e. having -ve deficiency

(i) Find the square of 6

Step 1: Base 10 and the deficiency is -4

Step 2: As 6 is 4 less than the base 10, we should further decrease it by its deficiency from the base i.e. 4 in this case. Therefore 6-4 = 2. This is the LHS of our answer.

2/

Step 3: The R.H.S. of the answer is obtained as the square of the deficiency. In this case $4^2 = 16$.

Step 4: Since the base is 10 only one digit is admissible on the R.H.S. and the extra digit we carry over as usual.

$2/_16 = 36$

Similarly we can obtain:

$$7^2 = (7 - 3)/3^2 = 4/9 = 49$$
$$8^2 = (8 - 2)/2^2 = 6/4 = 64$$
$$9^2 = (9 - 1)/1^2 = 8/1 = 81$$

Further we can extend the application of Yavadunam sutra to larger numbers

$$92^2 = (92 - 8)/8^2 = 84/64 = 8464$$
$$94^2 = (94 - 6)/6^2 = 88/36 = 8836$$
$$95^2 = (95 - 5)/5^2 = 90/25 = 9025$$
$$96^2 = (96 - 4)/4^2 = 92/16 = 9216$$
$$97^2 = (97 - 3)/03^2 = 94/09 = 9409$$
$$99^2 = (99 - 1)/01^2 = 98/01 = 9801$$
$$996^2 = (996 - 4)/04^2 = 992/016 = 992016$$
$$993^2 = (993 - 7)/07^2 = 986/049 = 986049$$
$$995^2 = (995 - 5)/05^2 = 990/025 = 990025$$
$$989^2 = (989 - 11)/011^2 = 978/121 = 978121$$
$$987^2 = (987 - 13)/013^2 = 974/(13+3)/3^2 = 16/9 = 169 = 974169$$
$$9989^2 = 9989 - 0011/0011^2 = 9978/0121 = 99780121$$
$$9993^2 = 9986/0049 = 99860049$$

12.2.2 CASE 2 The numbers greater than the base i.e. having positive deficiencies.

$$13^2 = 13 + 3/3^2 = 16/9 = 169$$

Since the deficiency is +ve, therefore, the number is increased by the amount of the deficiency.

$$16^2 = 16 + 6/6^2 = 22/{}_{3}6 = 256$$
$$18^2 = 18 + 8/8^2 = 26/{}_{6}4 = 324$$
$$20^2 = 20 + 10/10^2 = 30/{}_{10}0 = 40/0 = 400$$
$$104^2 = 104 + 4/04^2 = 108/16 = 10816$$
$$107^2 = 107 + 7/07^2 = 114/49 = 11449$$
$$112^2 = 112 + 12/12^2 = 124/{}_{1}44 = 12544$$
$$1011^2 = 1011 + 11/011^2 = 1022/121 = 1022/121 = 1022121$$

COMMENT: The algebraic proof of these methods have also been presented by Jagatguru Swami Sri Bharti Krisna Tirthaji Maharaja in his book on Vedic Mathematics.

12.3 THE SECOND COROLLARY

The second corollary is the Sutra Ekadhikena Purvena. This sutra is used as a subsutra in the present case. This is a special case of the first corollary, and is applicable for squaring the numbers having 5 as the last digit. It's meaning is by one more than the previous one.

12.3.1 Square of 35

Here the previous digit is 3. The sutra in this context tells us to multiply the previous digit (here 3) by one more than the previous number (3 + 1 = 4). Therefore, 3 x 4 = 12 is L.H.S. portion of the answer. The R.H.S. is simply the vertical multiplication product (5x5=25). Therefore $35^2 = 3 \times 4/5 \times 5 = 1225$.

12.3.2 Illustrations

$25^2 = 2 \times 3/25 = 6/25 = 625$
$45^2 = 4 \times 5/25 = 20/25 = 2025$
$65^2 = 6 \times 7/25 = 42/25 = 4225$
$85^2 = 8 \times 9/25 = 72/25 = 7225$
$75^2 = 7 \times 8/25 = 56/25 = 5625$
$95^2 = 9 \times 10/25 = 90/25 = 9025$
$115^2 = 11 \times 12/25 = 132/25 = 13225$

$$135^2 = 13 \times 14/25 = \begin{array}{l} 13+3 \\ \underline{14+4} \\ 18/2 \end{array} / \; 25 = 18225$$

$$145^2 = 14 \times 15/25 \quad \begin{array}{l} 14+4 \\ \underline{15+5} \\ 21/0 \end{array} / \; 25 = 21025$$

$195^2 = 19 \times 20/25 = 38025$

$155^2 = 15 \times 16/25 = 240/25 = 24025$

$185^2 = 18 \times 19/25 = 342/25 = 34225$

12.4 A sub-corollary to this is Antyayor-Dasakopi. This sub-sutra indicates that the above rule is also applicable to the multiplication of two numbers whose previous part is same and the sum of the last two digits is 10.

12.4.1 Illustration

Multiply 34 x 36

The previous part in both the numbers is same(3here) and the sum of last two digits is 10 (i.e. 4+6 = 10)

Therefore the Ekadhikena Purvena sutra is applicable.

Step 1 : The previous number is increased by 1 i.e. 3+1 = 4. And multiplied with the number. Therefore, 3x4=12 gives the L.H.S. portion of the answer.

Step 2: The R.H.S. is obtained by the vertical multiplication as before. Therefore, 4x6 = 24.

Step 3: So the answer is 34x36 = 12/24 = 1224

12.4.2 Further Illustration

86 x 84 = 8x9/6x4 = 72/24 = 7224

87 x 83 = 8x9/7x3 = 72/21 = 7221

73 x 77 = 7x8/3x7 = 56/21 = 5621

64 x 56 = 6x7/4x6 = 42/24 = 4224

104x106 =10x11/4x6=110/24 =11024

113x117 =11x12/3x7=132/21 =13221

Further, it is also applicable for the multiplication of numbers whose last digits together 100, 1000 etc.

193 x 107 = 1x2/93x7 = 20/651

693x607 = 6x7/93x7 =420/651

$$888\text{x}812 = 8\text{x}9/88\text{x}12 = 720/\begin{array}{l}88+\ \ 78\\ /\,\underline{12+\ \ \ 2}\\ 90/\,{}_{15}{}^{6}\end{array}$$

= 720/1056 = 721/056 = 721056

NOTE: Observe the fixation of extra zero on the right of L.H.S. portion.

12.5 THE THIRD COROLLARY

In mathematical astronomy we frequently require the multiplication of multiplier digits consisting entirely of nines with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena is used.

12.5.1 CASE I : Multiply 6 x 9

According to the sutra the multiplicand has to be decreased by 1 (this gives the LHS) and for the R.H.S. subtract the L.H.S. from the multiplier.(Get compliments of nines)

Therefore 6 x 9 = (6-1)/9-(6-1) = 5/4 = 54

Illustrations

(i) 9 x 9 = (9-1)/9-(9-1) = 9/1 = 81

(ii) 12 x 9 = (12-1)/9-11 = 11/$\bar{2}$= 10/10-2 = 10/8 = 108

(iii) 13 x99 = (13-1)/(99-12) = 12/ 87 = 1287

(iv) 19 x99 = 18/(99-18) = 18/ 81 = 1881

(v) 778 x999= 777/222 = 777222

(vi) 9863x999= 9862/ 0137 = 98620137

COMMENT: While obtaining the R.H.S. of the answer, by subtracting the L.H.S. from the multiplier(9999) all the digits including zero must be written (9999-9862 = 0137.

(vii) 3728387x9999999 = 3728386/6271613 =37283866271613

(viii) 12345678912x999999999=12345678911/87654321088
=1234567891187654321088

12.5.2 CASE II

If the multiplicand is having a smaller number of digits than the multiplier, the Sutra Ekanyunena is equally applicable. It is necessary to fill in the blanks on the left with the required number of zeros. The remaining process is unaltered.

Illustrations

(i) 8 x 99 = 08 x 99 = 07/(99-07) = 07/92 = 792

(ii) 67 x 999 = 067x999 = 066/ 933 = 66933

(iii) 737x99999 = 00737 x 99999 = 00736/99263 =

(iv) 83 x 9999999 = 0000083x9999999=0000082/9999917

= 829999917

12.5.3 CASE III (a) If the multiplier consists of smaller number of nines following the above procedure, the answer is obtained.

Illustrations

(i) $17 \times 9 = 16/9-16 = 16/\bar{7} = 15/10-7 = 15/3 = 153$

(ii) $163 \times 9 = 162/9 - 162$

$= 162/\overline{15^{3}} = 162 - 15/\bar{3}$

$= 147/\bar{3} = 146/10 - 3 = 1467$

Comments: This case yields vinculum on the R.H.S.

12.5.4 CASE III (b): The previous approach can be further shortened by the following steps illustrated and outlined below:

Step 1: Draw a line to divide the multiplicand into two parts. The RHS part having digits equal to the number of nines in the multiplier, and LHS having the excess digits.

Multiply 43 x 9

4/3 x 9

Step 2: Subtract one more than the excess portion (4+1) from the multiplicand. This gives the L.H.S. of the answer.

$$\begin{array}{r} 4\,/\,3 \times 9 \\ /\ \ -5 \\ \hline 3\,/8/ \end{array}$$

Step 3: The R.H.S. of the product is obtained by subtracting the right hand portion of the multiplicand by the Nikhilam rule. Therefore, the R.H.S. = 10 - 3 = 7

Therefore,

$$\begin{array}{r} 4\,/\,3\,/\ \ 9 \\ /\ -5/\ \ 3 \\ \hline 38/\ \ 7 = 387 \end{array}$$

12.5.5 Illustrations

(i) 73 x 9

```
7 3  x 9
7/3
 /-8 /03
----------
65 / 7      = 657
```

(ii) 132 x 9

```
13 /2
-1 / 4/   2
------------
11  8/8      = 1188
```

(iii) 122 x 99

```
  1 / 22  x99
   /  -2 /      -22
---------------------
    120/78   =      120/78 = 12073
```

(iv) 4389 x 99

```
43/89 /
 /-44/     89
-------------------
43 45/ 11                    = 434511
```

(v) 666778 x 9999

```
66/6778 /
     67/ 6778
-----------------
666711/ 3222
```

(vi) 111001 x 999

```
111/ 001 /
   / -112/001
-------------
110889/999
-------------
```

12.5.6: CASE III(C) In case of multiplication of numbers with lesser number of nines, the whole process can be still simplified by following the steps given here.

778 x 99

(i) The raw LHS is just one less than the previous number 777/

(ii) RHS digits are compliments of LHS digits from the nine (and equal to number of the nine digits).

$$777/_{7}22$$

Here 9 - 7 = 2; 9 - 7 = 2

To get the final LHS,

(iii) The excess digits of LHS are simply subtracted from the raw LHS obtained in (i)

OR we may say that the excess digits are written in vinculum form are carried to LHS.

$$777 + \bar{7}/22 = 77022$$

12.5.7 : Illustrations:

$$652 \times 99 = 651/_{\bar{6}}48$$

$$= 651 + \bar{6}/48 = 64548$$

$$3687 \times 999 = 3686/_{\bar{3}}313$$

$$= 3686 + \bar{3}/313 = 3683313$$

$$257642 \times 9999 = 257641/_{\overline{25}}2358$$

$$= 257641 + \overline{25}/2358$$

$$= 2576162358$$

वैदिक गणित अध्ययन

CHAPTER 13

THE GENERAL METHOD OF MULTIPLICATION

13.1 INTRODUCTION:

So far we have seen the use of the Vedic Sutras for the solution of specific types of problems. Now we shall learn the simple Vedic general method of multiplication. This method is based on the Urdhva-Tiryagvhyam Sutra. The simple meaning of this Sutra is vertically and crosswise. But this Sutra is very powerful and applicable for solving a large class of problems in mathematics. Further, it is interesting to point out that this brief Sutra has many fold applications. Prof. Nicholos and Prof. Williams have written a full book on applications of this Sutra (See news). The step by step process is outlined in the following sections:

13.2 SIMPLIFICATION:

For understanding the application of Urdhva Sutra i is important to first learn the method of simplification of different numbers of columns.

Level I: If there are only 2 digits in a single column we just multiply them vertically to get R the resultant number.

R = U1 x L1

1
: U
: L

For example

$$\therefore \quad R = 2 \times 3 = 6 \qquad \begin{array}{c} 2 \\ \underline{3} \end{array}$$

If however the resulting number consists of more than one digit, the extra digit at tenth place is written below the next higher space.

Level 2 : If the simplification consists of two columns with four digits the resulting number is obtained by cross multiplication of the diagonally opposite digits and algebraic addition of the products.

$$\begin{array}{ccc} 2 & 1 & \\ \cdot & \cdot & U \\ \cdot & \cdot & L \end{array}$$

U is the upper row

L is the lower row and U_1 and U_2 are the digits in the upper row in the 1st and 2nd column respectively. Similarly L_1 and L_2 are the digits of the lower row in column 1 and 2 respectively (from right).

Therefore $R = U_1 \text{ x } L_2 + U_2 \text{ x } L_1$

For example

$$\begin{array}{cc} 2 & 1 \\ 3 & 2 \\ \hline \end{array}$$

Therefore $R = 1 \text{ x } 3 + 2 \text{ x } 2 = 7$

Level 3: If we are simplifying three columns with six digits the result shall be obtained by cross multiplying the outer column digits (column 1 and 3) and adding s, we further add the product of the two digits of the central column(column 2)

$$\begin{array}{cccc} 3 & 2 & 1 & \\ \cdot & \cdot & \cdot & U \\ \cdot & \cdot & \cdot & L \end{array}$$

Therefore $R = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

For example

$$\begin{array}{ccc} 1 & 2 & 3 \\ 1 & 2 & 4 \\ \hline \end{array}$$

$R = 3 \text{ x } 1 + 4 \text{ x } 1 + 2 \text{ x } 2 = 11$

The extra digit (1) shall be written below the next higher place.

Level 4: If we are simplifying four columns with 8 digits the result should be obtained as the summation of the four cross multiplications. The first two shall consist of the cross multiplication of the outer most two columns (1 and 4 and the remaining two shall be the cross multiplication of the inner two columns (nos 2 and 3).

$$\begin{array}{ccccc} 4 & 3 & 2 & 1 & \\ \cdot & \cdot & \cdot & \cdot & U \\ \cdot & \cdot & \cdot & \cdot & L \end{array}$$

$R = U_1 x L_4 + U_4 x L_1 + U_2 x L_3 + U_3 x L_2$

For example

$$\begin{array}{cccc} 1 & 3 & 2 & 4 \\ 1 & 2 & 3 & 5 \\ \hline \\ \hline \end{array}$$

Therefore R = $4x1+1x5+2x2+3x3 = 22$

Level 5 : Similarly for 5 columns

$$\begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & \\ \cdot & \cdot & \cdot & \cdot & \cdot & U \\ \cdot & \cdot & \cdot & \cdot & \cdot & L \end{array}$$

$$R = (U_1 x L_5 + U_5 x L_1) + (U_2 x L_4 + U_4 x L_2) + (U_3 x L_3) \text{ etc.}$$

13.3 THE PROCEDURE:

After understading the method of simplifying different number of columns we can now learn the step by step process of applying 'Urdhva Sutra' with the help of the following examples.

Step 1: Simplify the first column. We start with the extreme left hand column. This gives the extreme left hand digit of the answer.

Step 2: We include the second column and simplify the two columns to get the second digit of the answer.

Step 3: Similarly we go on including one more column, in each step, and simplify to get the subsequent parts of the answer till the first column has operated upon all other columns. This would give us the result of the product upto the levels equal to the number of columns in the digits being multiplied.

Step 4: For subsequent parts of the answer, starting from the first column we go on eliminating one column at each stage, and simplify the remaining columns, till we are left with the extreme left column, simplification of which gives us the extreme left part of the answer.

Note: In each of these simplifications if the result consists of more than one digit number, the surplus digit is written below the next higher level.

13.4 ILLUSTRATION 1:

Let us consider the multiplication of 2 by 2 digit numbers.

Multiply 13 x 21

$$\begin{array}{r} 21 \\ \times\ \underline{13} \\ 3 \end{array}$$

Step 1: We simplify the first column

$R_1 = U_1 \times L_1$

$= 1 \times 3 = 3$

$R_1 = 3$ is the extreme right part of the answer and is written below the first column.

Step 2 : We operate the first column with each of the subsequent columns. We include the second column which on simpl fication gives the next digit

$R_2 = U_1 x L_2 + U_2 x L_1$

$= 1 \times 1 + 2 \times 3 = 7$

$R_2 = 7$ is the result at the next level and is written at the tenth level.

With this we complete the operation of the first colu

Step 3: We eliminate the first column and simplify the remaining columns (here only one column is left) therefore

$R_3 = U_2 \times L_2$

$= 2 \times 1 = 2$

$R_3 = 2$ is the resulting digit at the next level (hundred in this case). The result is written into appropria level.

$$\begin{array}{c} 2\ 1 \\ \underline{\times\ 1\ 3} \\ 2/7/3 \end{array} = 273$$

13.5 ILLUSTRATION 2:

Multiply 26 by 34.

$$\begin{array}{r} 34 \\ \times\ \underline{26} \\ \underline{/6/4} \\ \underline{2\ 2} \end{array}$$

Step 1 : We simplify the first column

$$R_1 = U_1 \times L_1$$

$$= 4 \times 6 = 24$$

$R_1 = 24$ the result is consisting of two digits. 4 is written at the unit place and the excess digit (here 2) is written below the next level.

Step 2: Include the second column and simplify

$$R_2 = U_1 \times L_2 + U_2 \times L_1$$

$$= 4 \times 2 + 3 \times 6 = 26$$

$R_2 = 26$ therefore 6 is written at the tenth place and excess digit (here 2) is written below the next level.

Step 3 : We eliminate the first column and simplify

$$R_3 = U_2 \times L_2$$

$$= 3 \times 2 = 6$$

$R_3 = 6$ is written at the next level.

Step 4: We add the carryover digits to the digits at the respective level

$$\begin{array}{r} 34 \\ \underline{\times\ 26} \\ \underline{6/6/4} \\ \underline{2\ 2\ \ } \\ \underline{8\ 8\ 4} = \end{array} \quad 884$$

13.6 ILLUSTRATION 3;

Multiply 123 by 425.

$$\begin{array}{r} 4\ 2\ 5 \\ \underline{\times\ 1\ 2\ 3} \\ \underline{1/6/5} \\ 2\ 1\ 1\ \ \ \ \ \end{array}$$

Step 1: Simplify the first column

$$R_1 = U_1 \times L_1$$

$$= 5 \times 3 = 15$$

$R_1 = 15$ therefore 5 is written in the unit column and excess digit 1 is written below the tenth column.

Step 2: We operate the first column with the subsequent columns, advancing column by column.

$R_2 = U_1 x L_2 + U_2 x L_1$

$= 5 x 2 + 2x3 = 16$

R_2 = 16 therefore 6 is written at the tenth place and excess digit is written below the hundredth column.

Step 3: We include the third column and simplify

$R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 5x1 + 4x3 + 2x2 = 21$

therefore 1 is written at hundredth place and the excess number 2 is written below the next level. This completes the operation of the first column on remaining columns.

Step 4: We eliminate the first column and simplify the remaining columns to compute the next part of the answer

$R_4 = U_2 x L_3 + U_3 x L_2$

$= 2x1 + 4x2 = 10$

R_4 = 10 therefore 0 is the entered in the thousandth place and excess digit 1 is written below the next level.

Step 5: For the next level we iliminate the second column and simplify the remaining columns. In the present example we are left with only the third column.

$R_5 = U_3 x L_3$

$= 4x1 = 4$

R_5 = 4 is entered at the next level.

```
   4 2 5
   1 2 3
---------
4/0/1/6/5
---------
1 2 1 1
---------
5 2 2 7 5   =   52275
```

13.7 ILLUSTRATION 4:

Multiply 7824 x 2638

```
   4      !
 ----------
   7824    U
 x 2638    L
 ----------
 /0/4/8/2
 ----------
10/9/2/3/
```

Step 1: Simplify the first column

$R_1 = U_1 \times L_1$

$= 4\times8 = 32$

$R_1 = 32$ therefore 2 is written in the unit column and excess digit 3 is written below the tenth column.

Step 2: We operate the first column with the subsequent columns, advancing column by column

$R_2 = U_1 x L_2 + U_2 x L_1$

$= 4 \times 3 + 2\times8 =$

$= 12 + 16$

$= 28$

$R_2 = 28$, therefore 8 is written at the tenth place and 2 below the hundredth place.

Step 3: Now include the third column

$R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 4\times6 + 8\times8 + 2\times3$

$= 24 + 64 + 6$

$= 94$

$R_3 = 94$; therefore, 4 is written at hundredth place and 9 below thousandth place.

Step 4: Including the fourth column

$R_4 = U_1 x L_4 + L_1 x U_4 + U_3 x L_2 + L_3 x U_2$

$= 4\times2 + 7\times8 + 8\times3 + 6\times2$

$= 8 + 56 + 24 + 12$

$= 100$

$R_4 = 100$ therefore 0 is written at thousandth place and the excess number 10 is written below the next level. This completes the operation of the first column on remaining columns.

Step 5: Eliminating the first column, we will simplify the remaining columns (from 2nd to 4th).

$R_5 = U_2xL_4 + U_4xL_2 + U_3xL_3$

$= 2x2 + 7x3 + 6x8$

$= 4+ 21 + 48$

$= 73$

$R_5 = 73$ therefore 3 is written at ten thousandth place and 7 below the next level.

Step 6 :
Further, eliminating the second column and simplifying the remaining two columns -

$R_6 = U_3xL_4 + U_4xL_3$

$= 8x2 + 7x6$

$= 16 + 42 = 58$

$R_6 = 58$ therefore 8 is written at next level that is lakh, and 5 below the million level.

Step 7: Now simplifying the last column, (eliminating 3rd column also).

$R_7 = U_4 \times L_4$

$= 7x2$

$R_7 = 14$ written at the ten lakhs place

```
        7 8 2 4
        2 6 3 8
 ------------------
 14/3/3/0/4/8/2
 ------------------
  5/7/10/9/2/3 /
 ------------------
   2/0 /6 /3 /9 /7 /1 /2     Ans.
```

13.8 ILLUSTRATION 5:

Multiply   2427 x 8882

```
     2 4 2 7
   x 8 8 8 2
   ---------
   /8/0/0/4
   ---------
 10/8/6/1/
   ---------
```

Step 1 : $R_1 = U_1 x L_1$

$= 7x2$

$= 14$

Step 2: $R_2 = U_1 x L_2 + L_1 x U_2$

$= 7x8 + 2x2$

$= 56 + 4 = 60$

Step 3: $R_3 = U_1 x L_3 + U_3 x L_1 + U_2 x L_2$

$= 7x8 + 4x2 + 2x8$

$= 56 + 8 + 16$

$= 80$

Step 4: $R_4 = U_4 x L_1 + U_1 x L_4 + U_3 x L_2 + L_3 x U_2$

$= 2x2 + 7x8 + 4x8 + 8x2$

$= 4 + 56 + 32 + 16$

$R_4 = 108$

Now the column elimination process is started.

Step 5: $R_5 = U_4 x L_2 + L_4 x U_2 + U_3 x L_3$

$= 2x8 + 2x8 + 4x8$

$= 16 + 16 + 32 = 64$

Step 6: $R_6 = U_3 x L_4 + U_4 x L_3$

$= 2x8 + 4x8$

$= 16 + 32 = 48$

Step 7: $R_7 = U_4 x L_4$

$= 2x8 = 16$

$$\begin{array}{l} \quad\quad\quad 2\ 4\ 2\ 7 \\ \quad\quad\quad 8\ 8\ 8\ 2 \\ \hline 16/8/4/8/0/0/4 \\ \hline 6\ 1\ 0\ 8\ 6\ 1 \\ \ \ \ 6 \\ \hline 1\ 7\ 5\ 5\ 6\ 6\ 1\ 4 \end{array} = 17556614$$

13.9 DISCUSSIONS

1. The 'Urdhava Sutra' gives us the key procedure for multiplying any two numbers. Once the method of simplification is properly understood and practised the remaining step by step procedure can be applied in a simple way. The whole process can be carried out through simple mental arthematic.

2. After a little practice even the excess carry over digit obtained at any level need not be written down below th next higher level but instead is directly added to the simplification value of the columns at the next level. If this approach is used the final answer can be obtained in one line.

3. In Vedic method we can also start computing the answer from the left hand side. This method shall be explaine in Pushp-2 along with the use of vinculum approach.

CHAPTER 14

MULTIPLICATION CHECK

वैदिक गणित अध्ययन

14.1 Introduction

In addition to the simplicity, speed and accuracy provided by the Vedic Mathematics for the multiplication process, it also has a unique feather in its cap. There is an extremely simple 'Vedic method' for checking the correctness of the multiplication results, using which even a middle standard student can check his calculations in less than 1 minute time.

14.2 Procedure

The very simple, superfast and accurate procedure is outlined in the following steps with the help of an example.

14.2.1 Example 1: Multiply 1006 x 1112

The 'Nikhilam Sutra' is invoked (Refer chapter 7.

Therefore

$$\begin{array}{l} 1112 + 112 \\ 1006 + 006 \\ \hline 1118 \;/\; 672 \quad = \quad 1118672 \end{array}$$

Step 1: Write down the multiplicand, multiplier and the result in three rows in the usual manner and draw a big square in front of it. Draw the two diagonals, thereby dividing the inner space into 4 triangles.

Step 2: Add the digits of the top number. If the addition exceeds 9, once again add the digits of the result of addition. Repeat this process till the final number is a single digit number.

In this example 1+1+1+2 = 5 is the resulting single number. Write this number in the top triangle.

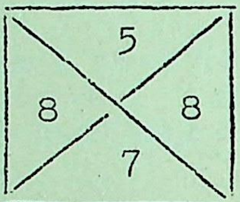
Step 3: Add the digits of the lower number i.e. the multiplier. If the addition exceeds 9, once again add the digits of the result of this addition. Repeat this process till a single digit number is obtained.

In the present example 1+0+0+6 = 7, the resulting single number has been obtained in single step.

Write this single digit number in the bottom triangle.

Step 4: Multiply the top single digit number in the top triangle with the single digit number in the bottom triangle. Write the result adjacent to the right hand side triangle. Add the digits of the resulting number. Write down this single digit number in the right hand side triangle.

$$\begin{array}{r} 1\ 1\ 1\ 2 \\ 1\ 0\ 0\ 6 \\ \hline 1118672 \\ \hline \end{array}$$



$5 \times 7 = 35$
$3 + 5 = 8$

Step 5: Add the digits of the total result of the multiplication. Once again if the addition exceeds 9, add these digits of the result of previous addition. Repeat this process till the final number is a single digit number. Write this single digit number in the empty space of left hand side triangle.

Step 6: CHECK

If the number in the left hand side triangle is equal to the number in the right hand side triangle. We can be reasonably sure that the multiplication result is correct.

In the present example, we get

$1 + 1 + 1 + 8 + 6 + 7 + 2 = 26$

$2 + 6 = 8$

We observe that the numbers in the right hand side triangle and the left hand side triangle are both equal to 8. This confirms the correctness of the multiplication.

NOTE: 1. This process may be further expedited by using the following steps :

(a) As the addition of the digits is being carried out, as soon as the addition exceeds 9, we can add this sub-result digits to transform them into a single digit number. We continue the further addition, using this single digit, as the sub-result.

In the present example 1 + 1 + 1 + 8 = 11. 11 is the two digit sub-result of addition of the left hand side 4 digits.

Therefore 1+1 = 2 is the simplified single digit sub-result. This will be used for further addition.

2 ÷ 6 = 8
8 + 7 = 15

Once again, at this stage, we get a two digit number (15) as a sub-result. This is reduced to a single digit. Therefore 1 + 5 = 6.

This sub-result is carried forward for subsequent addition. Therefore 6+ 2 = 8 is the final resulting single digit number obtained.

(b) In case there are any nines (9) in any of the numbers(the multiplicand, multiplier and the result), they can be simply eliminated during the addition process, without altering the final single digit result. This shall further simplify the already super simple process of checking. However, if any of these numbers consists entirely of nines and zeros, then only one nine should be retained, as the single digit result of the addition. Say for example the multiplier is 99090. Following this rule we can drop two nines and the resulting single digit number would be 9. Even if we add in the normal manner, we will get the same results.

9 + 9 = 18; therefore 1 + 8 = 9

9 + 0 = 0

9 + 9 = 18, therefore 1 + 8 = 9

9 + 0 = 9.

Quite obviously the 'Vedic Mathematics' procedure for checking of the multiplication is extremely simple wonderfully fast, yet quite accurate.

And at the top of it

Just 'a child's play'

14.3 Further Illustrations

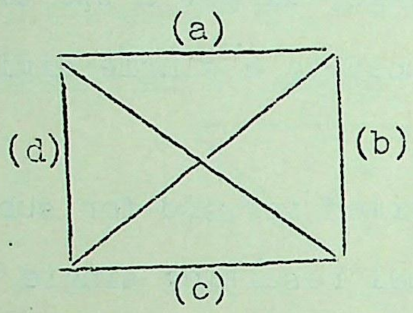
14.3.1 Example 2: Multiply 87965 x 99988

By applying Nikhilam we get the answer 8795444420

Step 1:

```
      87965
    x 99988
 ----------
 8795444420
 ----------
```

and draw a square with four equal triangles.



Step 2:

Add the digits of multiplicand to get one single digit number

8 + 7 = 15, 1 + 5 = 6;

6 + 9 = 6; 6 + 6 = 12, 1 + 2 = 3; 3 + 5 = 8

Enter 8 in the upper triangle (a).

Step 3: Add the digits of multiplier to get one single digit number

9 + 9 = 18, 1 + 8 =9; 9+9 = 9; 9 + 8 = 8;

We may leave all 9's and procede

8 + 8 = 16, 1 + 6 = 7;

Enter 7 in the lower triangle (c)

Step 4: Now multiply the digits of triangle's (a) and (c) i.e. 8 x 7 = 56 and convert this 56 into single digit 5+6 = 11, 1+1 =

Enter 2 in the R.H.S. triangle (b)

Step 5: Add the digits of the result to get one single digit nu

8 + 7 = 15, 1+5 = 6; 6+ 5 = 11, 1+ 1 = 2; 2 + 4 + 4 = 10,

1 + 0 = 1; 1 + 2 + 4 + 4 = 11, 1 + 1 = 2

Enter 2 in the L.H.S. triangle (d).

Step 6: Here 2 = 2. So our answer is correct.

14.3.2 Example 3: Multiply 1033 x 977

By applying Nikhilam we get the answer is 1009241. (OR) by the Urdhava sutra.

Step 1:

$$\begin{array}{r} 1033 \\ 977 \\ \hline 1009241 \\ \hline \end{array}$$

Draw a square with four equal triangle .

Step 2: Add the digits of multiplicand to get one single digit number

1 + 3 + 3 = 7

Enter 7 in the upper triangle(a).

Step 3: Add the digits of multiplier to get one single digit number.

9 + 7 + 7 = 14, 1 + 4 = 5

Enter 5 into the lower triangle(c).

Step 4: Now multiply the digits of the Triangle's (a) & (c) i.e. 7 x 5 = 35 and convert 35 into single digit 3 + 5 = 8.

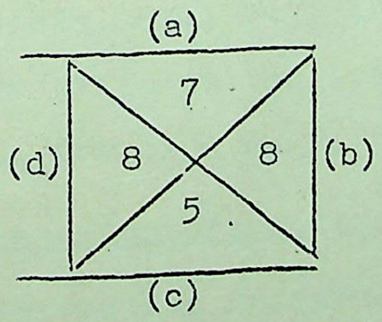
Enter 8 in the R.H.S. triangle.

Step 5: Add the digits of result to get one single number.

1 + 2 + 4 + 1 = 8 (Enter in the L.H.S. triangle (d).

Step 6: 8 = 8. So the answer is correct.

With practice the whole check can be applied mentally within a few seconds.



7 x 5 = 35;

3 + 5 = 8 .

CHAPTER 15

वैदिक गणित अध्ययन

RECURRING DECIMALS

15.1 INTROLUCTION:

The 'Ekadhikena Sutra' can also be used for computing the values of recurring numbers at magic speed. The Vedic method of computing the value of the recurring numbers is not only superfast but also very simple and easy. Here we shall just study some applications in specific type of problems. Further details about other types of decimals and fractions shall be taken up in latter volumes.

15.2 THE LAST DIGIT:

15.2.1 Fractions with last digit of 9.

First of all we shall take those fractions whose last digit of the denominator is 9; for example fractions like

$$\frac{1}{19}, \frac{1}{29}, \frac{1}{39}, \frac{1}{49} \text{ etc.}$$

We know that for every recurring decimal the last digit of the denominator when multiplied with the last digit of the recurring decimal will always yield a product ending in 9. In all the examples just mentioned the last digit of the denominator is 9. Therefore the last digit of the recurring decimals shall be 1 in all these cases. This property therefore gives us the last (right hand) digit of the recurring fraction as 1.

15.2.2 Denominators ending in other digits

For vulgar fractions whose denominators are 7, 13, 17 etc. we can easily establish the last digit of the decimal equivalent. We use the same property which tells us that the product of the last digit of the denominator and last digit of the decimal equivalent shall always end in 9. Therefore, the denominators ending in 7, 3 and 1 must necessarily yield decimals ending in [illegible] and 9 respectively (as 7x7 = 49, 3x3 = 9 and 1x9 = 9).

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

15.3 THE OPERATOR:

The operator key number for computing the value of the recurring decimal can be obtained by applying the simple 'Ekadhikena Sutra'. The simple meaning of the Sutra 'Ekadhikena Purvena' is 'by one more than the previous one'. Thus for $\frac{1}{49}$ the operator key is one more than the previous digit (4). Therefore operator is 4+1 = 5 similarly for $\frac{1}{39}$ the operator = 3+1 = 4, For $\frac{1}{29}$, operator = 2+1 = 3; For $\frac{1}{19}$, operator = 1+1 = 2 etc.

15.4 THE PROCEDURE:

'Ekadhikena Sutra' tells us that 'By one more than th previous one'. The first word' by is used both for multiplying and dividing. It is interesting to learn that this operator key can be used in both ways to compute the value of the recurring decimal. We shall first of all learn the method of using the operator key as a constant uniform multiplier.

15.4.1 Illustration 1: Let us take the fraction $\frac{1}{39}$.

As already explained the last right hand word digit shall be 1. Further, the operator key is 3+1 = 4. The subsequent, step by step procedure is outlined below.

Step 1: Put 1 as the extreme right digit and put a dot over it to indicate that it is the end of the set of recurring digits.

Step 2: We multiply this digit by the operator and write down the result towards the left hand of the previous digit. If the resulting number has more than one digit we put down the extra digit below the next level. In this case 1x4 = 4 is written on the L.H.S. of 1 (the previous digit).

$$4\ \dot{1}$$

Step 3: We multiply the previous digit(4) by the common constant operator key number (here 4). Therefore 4x4 = 16. We write down 6 on the L.H.S. of previous digit 4 and excess digit 1 in the following manner (below the next level).

$$\begin{array}{ccc} 6 & 4 & \dot{1} \\ 1 & & \end{array}$$

Step 4: We multiply the previous digit (6) by the operator 6x4 = 24 and add carry over digit 1 to yield the result 24+1 = 25. Once again we write 5 on L.H.S. of the previous digit 6 and excess digit 2 below the next level.

$$\begin{array}{ccccc} & 5 & 6 & 4 & \dot{1} \\ 2 & 1 & & & \end{array}$$

Step 5: We multiply the previous digit (5) by the operator, 5x4 = 20 and add carry over digit 2 to the result 20+2=22. The result is written as before.

$$\begin{array}{cccccc} & 2 & 5 & 6 & 4 & \dot{1} \\ 2 & 2 & 1 & & & \end{array}$$

Step 6: We multiply the previous digit 2 by the operator(4), therefore 2x4 = 8 and add the carry over digit 2 to 8, so 2+8 = 10. So our result will become

$$\begin{array}{ccccccc} . & 0 & 2 & 5 & 6 & 4 & \dot{1} \\ 1 & 2 & 2 & 1 & & & \end{array}$$

Step 7: We multiply 0 x 4 = 0 and add the carry over digit 1 so 0+1 = 1etc. So we find that we have started getting the same digits after 0. Therefore we can put a dot over 0 to indicate that the set of recurring decimals starts from 0. The decimal is placed before 0. Thus the set of recurring decimal is

$$\frac{1}{39} = 0.\dot{0}2564\dot{1}$$

Discussion: Since 39 is a prime number, the behaviour of $\frac{1}{39}$ is different from other recurring numbers. Let us consider the example of the recurring number $\frac{1}{29}$. As our next illustration.

15.4.2 Illustration 2: Evaluate $\frac{1}{29}$.

Step 1: Here again since the last digit of the denominator is 9, the last digit of the decimal equivalent shall end in 1.

Step 2: The operator key by Ekadhika Sutra is equal to 'One more than the previous digit'. Here 2+1 = 3 is the constant, uniform multiplier.

Step 3: The operation; (i) Put 1 as the last right digit (ii) Multiply the previous digit by the operator key 1x3=3 and write the result before the previous digit (1). 31

(iii) Multiply the previous digit (3) by the operator (3) an write down before the previous digit (3), therefore 3x3=9.

$$93\dot{1}$$

(iv) Further multiply the previous digit(9) by the operator (3) 9x3=27 this results in a two digit number. As already explained we write down the unit place digit (7) before the previous digit (9) and the excess carry over digit (2) is written below the next level.

$$_{2}793\dot{1}$$

(v) Now we multiply 7 with operator 3 and get 7x3 = 21. But since there is 2 carry over already written below, we add it to the product obtained. Therefore 21+2 = 23. Once again the resulting number has two digits and the excess digit is written below next place.

$$\begin{array}{l} 3793\dot{1} \\ 22 \end{array}$$

(vi) Similarly we go on computing the remaining digits till the digits start repeating. In case of $\frac{1}{29}$ we shall get 28 digits as the result but in Vedic method the property of numbers is so thoroughly understood so that even this simple operation of multiplying 28 times need not be carried out. There is a simpler way of writing down the remaining half of the result, after the first half has been computed. As soon as we obtain the result equal to the difference of the denominator and the numerator, we can be sure that half of the result has been just obtained with the previous digit. Therefore we put a slash just before the previous digit to indicate that half of the resulting digits have already been written. We can straight a way write down the remaining digits towards the left hand side of slash by simply writing the compliments of existing digits from 9. In the present example we obtain 19 as the 14th product. Therefore 9 is written as the 14th digit and excess digit 1 is written below the next level. Now when we multiply 9 with 3 we get 9x3=27 and add the carry over 1, 27+1 = 28 this result is equal to 29-1. Therefore we put a slash before 9. The digits on the L.H.S. of slash are obtained by simply subtracting, the corresponding digits already obtained, from 9. Therefore 9-1=8 is the 15th digit, 9-3= 6 is the 16th digit, 9-9=0 is the 17th digit etc. and finally 9-9 =0 is the 28th digit. Therefore we put a dot over 0 and a decimal point before it.

$$\begin{array}{l} .\dot{0}34482758620\underline{68}/96551724137 93\dot{1} \\ \qquad\qquad\qquad\quad 21\ \ 11\ \ 2\ \ 1\ \ 122 \end{array}$$

15.5 Illustration 3: Let us take vulgar fraction not ending in 9. Say $\frac{1}{7}$.

Step 1: We put down $\frac{1}{7}$ as $\frac{1}{7} \times \frac{7}{7} = \frac{7}{49}$.

Step 2: The operator here by 'Ekadhika Sutra' is 4+1= 5.

Step 3: Here our starting digit is 7.

Step 4: We multiply the previous digit (7) by the operator (5) and get 35. 5 is written before the previous digit and three below the next number.

Step 5: Next multiply 5 by 5 and add the carry over 3 to it. We get 28.

Step 6: Now multiply 8 by 5 and add 2 we get 42. But denominator-numerator (49-7) is also 42. Hence we stop the calculation and put a slash before the already obtained three digits.

2 /8 5 7
2/ 3

Complete the remaining digits by obtaining the compliments from 9 of already obtained three digits. Therefore 9-7=2 is the 4th digit, 9-5 = 4 is the 5th digit and 9-8 = 1 is the 6th digit. Hence our answer is $\frac{1}{7} = .\dot{1}4285\dot{7}$

Note: Starting from the left hand side digit, the same result would be obtained if we start dividing numerator by the operator key. In this case of division we can stop at any level desired.

Discussions:

1. In this chapter we have studied very interesting, easy and simple applications of 'Ekadhika Sutra'.
2. The speed with which the computations of recuring numbers are done looks almost like a magic [illegible] compared with the present long, tedious and time consuming method.
3. Further, this system is very simple and easy.
4. In this case also the Vedic method leaves behind the electronic calculators as the ordinary calculators shall not be in a position to compute more than 10 to 12 digits.
5. Further, the vedic methods for any division, factorisation, solution of higher order equations are equally fast, simple and easy, and shall be dealt with in later volumes.
6. In addition to these methods of solving varied class of problems the Vedic methods for checking the correctness of answer obtained, are also available.

CHAPTER-29

DIVISION - BY NIKHILAM METHOD

29.1 INTRODUCTION:

We have seen how NIKHILAM Method can be used for the multiplication of big numbers.Some other applications of the NIKHILAM SUTRA in VINCULUM Numbers, Table etc.have also been discussed. It is interesting too learn that the Nikhilam Sutra can also be used for the division. Like multiplication the Nikhilam Method is most convenient when the divisor consists of the big digits. Bigger the digits easier is the solution by NIKHILAM Method.

29.2 The NIKHILAM METHOD : As a first step we modify the diviser by using the NIKHILAM SUTRA and than use the modified divisor for subsequent simple operations which never involve any type of division. The step by step procedure is further illustrated with the help of following illustrations.

Type-I:

29.2.1 ILLUSTRATION 1

Divide 34 by 9

9 ÷ 34

Step 1 - We modify our divisor by using Nikhilam Sutra, Namely, the last digit is subtracted from 10 and next from nines. This modified diviser so obtained will be used for subsequent operation.

9 ÷ 34
1

Step 2: We write modified diviser(MD) and dividend(D) as follows

9 ÷ 34
1

Now we draw a slash in the dividend (D) after the number of digits equal to the number of the digits in the divisor. Here MD and D digits are numbered from LHS and RHS respectivel Since in this example we have only 1 digit divisor. We shall place an oblique line just after 1 digit of D. Thus column C

1st 2nd 1st
9 ÷ 3/4
1

Step 3: Now multiply the last digit of D (i.e. in the appear at 2nd column in the present case) with the last digit of MD (i.e. the one at the first and last place in the present case) and place the product just below the previous column of D (which is the 1st column in this example).

$$\begin{array}{l} 9 \div 3/4 \\ 1 \quad\;\; 3 \\ \hline \end{array}$$

In the present example we have already reached the first digit of the dividend and hence no subsequent operations of MD are required.

Step 4 : Now draw a line below the second Row and sum the digits(column wise) appearing below 1st column , 2nd column and so on. The answer so obtained will have two parts. First part is on R.H.S. of oblique line. It is Remainder R. Second appearing along L.H.S. of oblique line. This is the quotient.

$$\begin{array}{l} 9 \div 3/4 \\ 1 \quad\;\; 3 \\ \hline \quad\;\; 3/7 \end{array}$$

Here 3 is Quotient and
7 is Remainder.

29.2.2 ILLUSTRATION 2:

Divide 60 by 9

9 ÷ 60

Step 1: Using NIKHILAM SUTRA we modify our divisor.
Here MD = 1

Step 2: We write MD and D as

9 ÷ 60
1

Step 3: We draw a slesh after 1 digit of D and number MD and D

2nd 1st
9 ÷ 6 / 0
1

Step 4: Multiply the last digit of D with the digit of MD and product is placed below the previous digit (1^{st} column)

2nd 1st
9 ÷ 6 / 0
1 6

Step 5: No subsequent operation is required.

Step 6: Draw the line below the second row and add the 1^{st} and 2^{nd} column digits, we get result.

2nd 1st
9 ÷ 6/0
6
―――――
6/6

Hence

Quotient = 6
Remainder= 6

2.9.2.3 ILLUSTRATION 3 Divide 71 by 9

9 ÷ 71

Straightaway we can write the solution following by previously elaborated steps mentally.

9 ÷ 7/1
1 7
―――――
7/8

Therefore R = 8
Q = 7

29.2.4 ILLUSTRATION 4

Similarly divide 53 by 9

9 ÷ 5/3
1 5
―――――
5/8

Therefore R = 8
Q = 5

29.2.5 ILLUSTRATION 5

Now If we have bigger dividend the process is exactly the same with only one small additional step.

Divide 213 by 9

9 ÷ 213

Step 1: Using Nikhilam Sutra we modify our divisor. Here MD =

Step 2: We write MD and D as

9 ÷ 213
1

Step 3: We draw a slash after 1 digit of D and number MD from RHS. and product is placed below the previous digit (2nd column)

$$\begin{array}{llll} & 3^{rd} & 2^{nd} & 1^{st} \\ 9 \div & 2 & 1 / & 3 \\ 1 & & 2 & \end{array}$$

Step 4: Multiply the last digit of D with the digit of MD and product is placed below the previous digit (2nd column).

$$\begin{array}{ll} 9 \div & 2\,1/3 \\ 1 & \underline{\;\;2\;\;} \\ & 2\;3 \end{array}$$

Step 4(a): Now after adding the digits of 2nd Column we get 1 + 2 = 3; which is the new operator.

Step 5: Again multiply this operator i.e. 3 and MD digit i.e.1 and the product (i.e. 1x3=3) is placed below the previous digit (in the 1st column)

$$\begin{array}{ll} 9 \div & 2\,1/3 \\ & \;\;2/3 \end{array}$$

Step 5(a): On adding the digits of 1st column we get the result because we have reached the last digit of D

$$\begin{array}{ll} 9 \div & 2\,1/3 \\ & \;\;2\;3 \\ \hline & 23/\;6 \\ \hline \end{array}$$

Hence Q = 23
R = 6

29.2.6 <u>ILLUSTRATION 6</u>: Divide 323 by 9

Following the previously explained steps mentally we can perform the division straightway as given below:

```
9 ÷  32/3
1     3
        5
    35/8
```

Hence Q = 35
R = 8

29.2.7 <u>ILLUSTRATION 7</u>:

Even if the number of digits in the dividend are still larger we can get the quotient and the remainder with equal ease.

Divide 1 2 1 2 1 by 9

```
9 ÷ 1 2 1 2 1
```

<u>Step 1</u>: Using NIKHILAM MD = 1

<u>Step 2</u>: We write MD,2 D as

```
9 ÷ 1 2 1 2 1
```

<u>Step 3</u>: We draw a slash after 1 digit of D and number D from R.H.S.

```
      5th 4th 3rd 2nd 1st
9 ÷ 1   2   1 12 /2/  1
1
```

<u>Step 4</u>: Multiply the last digit of D,i.e. 1 with the digit of D MD,i.e. 1 and product, i.e. 1x1 =1, is placed below the previous digit (4th column)

```
9 ÷ 1 2 1 2/1
1     1
    -----------
    1 3
```

<u>Step 4(a)</u> : Now adding the digits of 4th column we get the new operator as 2+1 =3.

<u>Step 5</u>: Again multiply this operator, i.e. 3, with MD digit i.e. 1, and product is placed below the previous digit

(3rd column).

9 ÷ 1 2 1 2/1
1   1
        3

Step 5(a): Again adding the digits of 3rd column we get the next operator as 1 + 3 = 4.

Step 6 : Further multiply the operator 4 with MD digit 1 and product is placed below the 2nd column

9 ÷ 1 2 1 2/1
1   1
        3
          4
   1 3 4 6

Step 6(a) Further adding the 2nd column,we get the next new operator, which is 4 + 2 = 6.

Step 7: Lastly we multiply this operator 6 with MD digit 1 and product is placed below the previous digit (1st column).

9 ÷ 1 2 1 2/1
1   1 3 4 6

Step 7(a): After adding the digits of all the columns we get a result as

9 ÷ 1 2 1 2/1
1   1 3 4 6
   1 3 4 6/7

Hence Q = 1346
R = 7

29.2.8 ILLUSTRATION 8:

Divide 1 1 2 0 2 1 1 by 9

9 ÷ 1 1 2 0 2 1 /1
1   1
        2
          4
            4
              6
                7
1 2 4 4 6 7 /8

Hence Q = 1 2 4 4 6 7
R = 8

Notes:- 1) In all these division problems we have obtained the result without doing any division whatsoever(2) we have only been multiplying a single digit by another single digit. and adding to get the result. (3) In the previous examples since our MD was 1, there was really no multiplication at all.

29.3 TYPE 2:

We shall consider the second type of problems.

29.3.1 ILLUSTRATION 9

Divide 1012 by 8

```
8 ÷ 1 0 1 2
```

Step 1 : Using NIKHILAM SUTRA we modify the divisor as MD=
(here

Step 2 : We write as(here) 8 ÷ 1 0 1 2

```
8 ÷ 1 0 1 2
2
```

Step 3 : We draw a slash after 1 digit of D and number the digits of D from R.H.S.

```
    4th 3rd 2nd 1st
8 ÷ 1   0   1  / 2
2
```

Step 4: Multiply the last digit of D i.e. 1 with the MD digit 2 and product (1 x 2 = 2) is placed below the previous digit (3^{rd} column), as before.

```
8 ÷ 1 0 1/2
      2
  -------
    1 2
```

Step 4(a): On adding the digits of 3^{rd} column we get a new operator as 0 + 2 = 2 (operator)

Step 5: Multiply this operator i.e. 2 with MD digit i.e. 2 and product (2x2 = 4) is placed below the previous digit (2nd column).

```
8 ÷ 1 0 1/2
2     2
        4
  ---------
    1 2 5/
```

29-8

Step 5(a): Again adding the digits of 2^{nd} column we get next operator as 1+4 = 5(operator).

Step 6: Further multiply this operator(5) with MD digit(2) and product is placed below the previous digit (1^{st} column)

8 ÷ 1 0 1/2
2   2 4/10
———————
1 2 5/

Step 6(a): Further adding the digits of 1st column we reach at the stage where we find that the remainder is bigger than the divisor

8 ÷ 1 0 1/2
       2 4/10
———————
1 2 5/12

NOTE:- We find that the remainder so obtained is bigger than the divisor(8). Therefore we have to modify the remainde and carry the additional 1 to the quotient side. So that 12 - 8 = 4 is the final remainder and 125+1 = 126 is the fina quotient. This can also be done mechanically by repeating the same NIKHILAM METHOD and this time we divide the Remainder as shown below:

8 ÷ 1 0 1 /2
       2 4 10
———————
1 2 5/1/2
            2
———————
1 2 5/1/4

OR

126/4

Therefore Q = 126
R = 4

2.9.3.2 ILLUSTRATION 10

Divide 67 by 9

9 ÷ 6 7

Step 1: Using NIKHILAM SUTRA we modify our divisor.
MD = 1 (here)

Step 2: We write MD and D as

$$\begin{array}{l} 9 \div 6\ 7 \\ 1 \end{array}$$

X2

Step 3: We draw a slash after 1 digit of D and numbered MD and D

$$\begin{array}{lcc} & \text{2nd} & \text{1st} \\ 9 \div & 6\ / & 7 \\ 1 & & \end{array}$$

Step 4: Multiply the last digit of D with the last digit of MD and the product is placed below the previous digit (1st column)

$$\begin{array}{lcc} & \text{2nd} & \text{1st} \\ 9 \div & 6\ / & 7 \\ 1 & & 6 \end{array}$$

(ब) No subsequent operation is required

Step 5: Draw the line below the second row and add the 1st and 2nd column digits we get result

$$\begin{array}{lcc} & \text{2nd} & \text{1st} \\ 9 \div & 6\ / & 7 \\ 1 & & 6 \\ \hline & 6\ / & 13 \end{array}$$

So the Remainder is 13 - 9 = 4
and the Quotient is 6 + 1 = 7
directly

$$\begin{array}{lcc} 9 \div & 6\ / & 7 \\ & & 6 \\ \hline & 6\ / & 1\cancel{3}3 \\ & & 1 \\ & 6\ / & 1/4 = 7/4 \end{array}$$

29.3.3 ILLUSTRATION 11:

Divide 8 9 by 9

$$9 \div 8\ 9$$

$$\begin{array}{lcc} 9 \div & 8\ / & 9 \\ \cancel{1} & & 8 \\ \hline & 8\ / & 17 \end{array}$$

OR 8/17 - 9
OR 9/8

Hence Remainder = 8
Quotient = 9

Directly
9 ÷ 8/9
8
8/17
8/1/7
1
8/1/8
9/8

2·
2.9.3.4 ILLUSTRATION 12

Divide 167 by 9
9 ÷ 167

Directly
9 ÷ 16/7
1 1 7
17/1/4
1
18/5

So Quotient is 18
Remainder is 5

29.3.5 ILLUSTRATION 13

Divide 25642 by 9

9 ÷ 2 5 6 4 2

9 ÷ 2 5 6 4 / 2
2 7 13/17
2 7 3 7 /19
1 1

OR

2 8 4 7 /19 OR 1/9
1/1/0
2/1

Hence

2847 + 2/1
2849/1

So Quotient = 2849
Remainder= 1

29.3.6 ILLUSTRATION 14: Divide 1011638 by 9

$$\begin{array}{r} 9 \div 1\ 0\ 1\ 1\ 6\ 3\ /\ 8 \\ 1\ 1\ 2\ 3\ 9\ /12 \\ \hline 1\ 1\ 2\ 3\ 9_{1}2/\ 20 \end{array}$$

Add the digits of 1st column

$$\begin{array}{r} 2/0 \\ 2 \\ \hline 2/2 \end{array}$$

The Quotient is

$$\begin{array}{r} 1\ 1\ 2\ 3\ 9\ 2/\ /2 \\ 11/2 \\ \hline 1\ 1\ 2\ 4\ 0\ 2/\ /2 \\ 2 \end{array}$$

OR 1 1 2 4 0 4/2

The Remainder is 2

Notes:(1) All the rows have been converged into single row in order to save space.

(2) Here in 1st instance, we get remainder as 20 which is still greater than divisor. Hence divison has been further extended and the resulting quotient digit i.e. 2 has been added to previous quotient.

29.5 TYPE 3:

If divisor has more than one digit the Nikhilam method still works with equal ease

29.4.1 ILLUSTRATION 15:

Divide 216 by 89

$89 \div 216$

Step 1: Using NIKHILAM We modify the divisor

Here MD = 1 1

Step 2: We write as

$$\begin{array}{l} 89 \div 216 \\ 11 \end{array}$$

Step 3: We draw a slash after 2 digits and number of the digits of the Dividend from R.H.S. and the digits of Divisor numbered from L.H.S.

$$\begin{array}{rcccc}
 & \text{3rd} & & \text{2nd} & \text{1st}\\
8\ 9 \div & 2 & / & 1 & 6\\
1\ 1 & & & & \\
\text{1st 2nd} & & & &
\end{array}$$

Step 4: Multiply the last digit of D with the first and second digits of MD respectively and place the product below the previous digits (2nd and 1st column respectively) as follows

$$\begin{array}{rcccc}
8\ 9 \div & 2 & / & 1 & 6\\
11 & & & 2 & 2\\
\hline
 & 2 & / & &
\end{array}$$

Step 5: On adding the digits of 2nd and 1st column we get result.

$$\begin{array}{rcccc}
8\ 9 \div & 2 & / & 1 & 6\\
111 & & & 2 & 2\\
\hline
 & 2 & / & 3 & 8
\end{array}$$

So Quotient = 2
Remainder=38

29.4.1 ILLUSTRATION 16:

Divide 112 by 87

$$\begin{array}{rccc}
87 \div & 1 & / & 12\\
13 & & & 13\\
\hline
 & 1 & / & 25
\end{array}$$

Hence Result
Q = 1
R = 25

29.4.3 ILLUSTRATION 17:

Divide 3 4 5 6 7 by 89

89 ÷ 3 4 5 6 7

Step 1: We modify the divisor using NIKHILAM
MD = 11

Step 2: We write as

$$89 \div 3\ 4\ 5\ 6\ 7$$

Step 3: We draw a slash after 2 digits of the D and numbered the digits of D from R.H.S. respectively but number l the digits of Divisor from L.H.S. as

$$\begin{array}{llllllll}
 & & & 5^{th} & 4^{th} & 3^{rd} & 2^{nd} & 1^{st} \\
 8 & 9 & \div & 3 & 4 & 5\,/ & 6 & 7 \\
 1 & 1 & & & & & & \\
 1^{st} & 2^{nd} & & \hline 3 & & & &
\end{array}$$

Step 4: Now multiply the last digit of D i.e. 3 with 1^{st} and 2^{nd} digit respectively (i.e.1 and 1) and the product is placed below the previous digits (4^{th} and 3^{rd} column respectively) as

$$\begin{array}{lllllll}
 8\ 9 & \div & 3 & 4 & 5\,/ & 6 & 7 \\
 111 & & & 3 & 3 & & \\
 \hline & & 3 & & & &
\end{array}$$

Step 4(a) : On adding the digits of 4^{th} column we get a operator Here 4 + 3 = 7 which is new operator.

Step 5 : Multiply this operator 7 with the MD digits 1 and 1 respectively and place the product below the 3^{rd} and 2^{nd} columns respectively

$$\begin{array}{lllllll}
 8\ 9 & \div & 3 & 4 & 5\,/ & 6 & 7 \\
 1\ 1 & & & 3 & 3 & & \\
 & & & & 7\,/ & 7 & \\
 \hline & & 3 & 7 & 15\,/ & &
\end{array}$$

Step 5(a): After adding the digits of 3^{rd} column we get a next operator i.e. 5 + 3 + 7 = 15

Step 6: Further multiply the operator 15 with MD digits 1 and 1 respectively and placed the produce below the 2^{nd} and 1^{st} column respectively.

$$\begin{array}{lllllll}
 8\ 9 & \div & 3 & 4 & 5\,/ & 6 & 7 \\
 1\ 1 & & & 3 & 3 & & \\
 & & & & 7\,/ & 7 & \\
 & & & & /& 15 & 15 \\
 \hline & & 3 & 7 & 15 & &
\end{array}$$

Step 6(a) : Further adding the digits of all the columns

$$\begin{array}{lllll}
8\ 9 \div & 3 & 4 & 5/6 & 7 \\
1\ 1 & & 3 & 3 & \\
 & & & 7/7 & \\
 & & & /15 & 15 \\
\hline
 & 3 & 7_1 & 5/_2 8 & {}_2{}^2 \\
 & 3 & 8 & 5/3/0 & 2 \\
 & & & & 33 \\
\hline
 & 3 & 8 & 5/3/3 & 5 \\
 & 3 & 8 & 8/3 & 5
\end{array}$$

29.4.4 ILLUSTRATION 18

Divide 1 0 111 6 3 8 by 9 8 7

9 8 7 ÷ 1 0 1 1 6 3 8

Step 1: Using Nikhilam Sutra first we modify the given divisor (and get 013).

9 8 7 ÷ 1 0 1 1 6 3 8
0 1 3

Step 2: Here we have 3-digit divisor so we put a oblique line just after 3-digits from the R.H.S. of D

9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3

Step 3: Now the last digit of D shall be the operator which shall be used as a multiplier, and multiplied with all the digits of the MD (from Left) and the product shall be placed below the subsequent digits of D as before. Here the last digit of D is 1, therefore 1 is our multiplier. We multiply 1 with 0, and write the product 0 just below the previous digit (6th column)

	7th	6th	5th	4th	3rd	2nd	1st
9 8 7 ÷	1	0	1	1 /	6	3	8
0 1 3		0					
1st 2nd 3rd							

(b) Again multiply it by 2nd digit of MD and place the product below the previous digit (5th column

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1
```

(c) Further multiply with 3^{rd} digit of MD and place the product below the 4th column:

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1 3
```

<u>Step 4</u>: Now for the next step of multiplication an operator to be multiplied is obtained by summation of the previous digit (6th column).

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1 3

        ------
        1 0
```

So we get operator 0 (a) This 0 is multiplied by the last digit of MD and product is placed below the 5th column in 3rd row.

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1 3
            0
```

(b) Multiplication with 0 will now be repeated with 2^{nd} 1^{st} digit of MD and their product shall be placed below 4th and 3rd column respectively

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1 3
            0 0 0
        ----------
        1 0 2
```

<u>Step 5</u> : On adding 5th column we will get(which is 2 here)operator This 2 is multiplied by the last digit of MD and product is placed below the 4th column in 4th row.

Multiplication with 2 will now be repeated with 2nd and 1st digits of MD and their product shall be placed below 3rd and 2nd column respectively:

```
9 8 7 ÷ 1 0 1 1/6 3 8
          0 1 3/
            0 0/0
              0/2 6
        -----------
        1 0 2 4/
```

Step 6: Now on adding 4th column again we get operator as 4. Now multiply operator 4 with the 3^{rd}, 2^{nd} and 1^{st} digit of MD and place the products below the 3^{rd}, 2^{nd} and 1^{st} column and hence our result is obtained

```
9 8 7 ÷ 1 0 1 1/6 3 8
0 1 3     0 1 3
            0 0 0
              0 2 6
                0 4 12
        ---------------
        1 0 2 4/8 3 0
                 1 2
```

We note that the number of rows of the product is equal to the number of digit of D appearing after the oblique line drawn in step 2.

Step 7: Now Remainder is obtained on adding the carries i.e. 950

```
 8 3 0
 1 2
-------
 9 5 0
-------
```

and our Quotient is

Q = 1 0 2 4

R = 9 5 0

29.4.5 **ILLUSTRATION 19**: Divide 20135 by 9818

```
9 8 1 8 ÷ 2/0 1 3 5
0 1 8 2     0 2 16 4
          -----------
          2/0 3 9 9
               1
     or   2/0 4 9 9
```

Here the M.D. contains 8 which is a bigger digit. It could be modified by using Vinculum approach. However the products will also appear with negative sign and their subsequent con- addition will be algebrically done as explained below.

$$\begin{array}{ll} 9\;8\;1\;8 \div & 2/0\;1\;3\;5 \\ 0\;1\;8\;2 & \quad 0\;4\;\bar{4}\;4 \\ 0\;2\;\bar{2}\;2 & \\ \cline{2-2} & 2/0\;5\;\bar{1}\;9 \end{array}$$

Note:- The operation becomes still simpler by using the Vinulutuon approach.

On Normalising 2/0499. Hence Q = 2

R = 0499

29.4.6 ILLUSTRATION 20

Divide 300000 by 9888

$$\begin{array}{ll} 9\;8\;8\;8 \div & 3\;0/0\;0\;0\;0 \\ 0\;1\;1\;2 & \quad\;\; 0\;/3\;3\;6 \\ & \qquad\;\; /0\;0\;0\;0 \\ \cline{2-2} & 30/3\;\;3\;6\;0 \end{array}$$

Q = 30

R = 3360

Note: Zeros in M.D. are never ignored whenever these appear.

29.4.7 ILLUSTRATION 21

Divide 98564318 by 9886

Here some of the Dividend digits are greater than 5. Hence the dividend is modified by Vinculum approach. Thus

$$\begin{array}{ll} 9\;9\;8\;6 \div & 9\;8\;5\;6\;4\;3\;1\;8 \\ 9\;9\;8\;6 \overset{6}{\div} & 1\;0\;\bar{1}\;\bar{4}\;\bar{4}/\;4\;3\;2\;\bar{2} \\ 0\;0\;1\;4 & \;\;0\;0\;1\;4 \\ & \quad\;\; 0\;0\;0\;0 \\ & \qquad\;\; 0\;0\;\bar{1}\;\bar{4} \\ & \qquad\quad\; 0\;\;0\;\bar{3}\;\overline{12} \\ & \qquad\qquad\;\; 0\;0\;0\;0 \\ \cline{2-2} & 1\;\bar{0}\;1\;\bar{3}\;0/\;3\;\bar{4}\;\overline{10}\;\bar{2} \\ \text{OR} & 1\;0\;\bar{1}\;\bar{3}\;0/\;3\;\bar{5}\;0\;\bar{2} \end{array}$$

OR 9 8 7 0/2 4 9 8

Q = 9870 R = 2498

Note: It may be noted that $\overline{12}$ in 4th row obtained by multiplying $\overline{3}$ by 4 is placed in 2^{nd} column of dividend while summing $\overline{2}$ of $\overline{12}$ is added in 2nd column and remaining $\overline{1}$ of $\overline{12}$ is carried over to next column.

Notes: (1) The operations of multiplications may yield more than one digit. In that case, entire product is to be placed in the position columnwise as above. No shifting of columns is permissible in such cases.

(2) While summing columnwise, carryovers are possible. But at no stage carry over from remainder side is to be done to the quotient side. In such cases the remainder digits may be more than the columns appearing before oblique line.

29.5 DISCUSSIONS:

(1) In all these examples we have seen that there is absolutely no need of division at all. As such the use of the trial digits as require in the present division process is eliminated.

(2) The whole process of division by even real big numbers (see illustrations 16, 18, 19 and 21) is converted into simple series of multiplications and simple additions. Even these multiplications ~~are~~ of a one digit number by another single digit, at a time.

(3) The work tends to become a little more difficult if the dividend has bigger digits but this difficulty can also be reduced by using the VINCULUM APPROACH (as illustration 21).

(4) Infact in the vedic method of division by NIKHILAM SUTRA we perform no division, no subtraction and use only small single digit multiplication and addition which is much simpler than the present system.

29.6 Further ILLUSTRATIONS: A few, typical problems are further solved the following sections before taking up the problems of getting answer in the decimal form.

29.6.1 ILLUSTRATION 22: Divide 9876534201 by 8876

8 8 7 6 ÷ 9 8 7 6 5 3 4 2 0 1

First we change the dividend into vinculum form

8 8 7 6 ÷ 1 0 $\bar{1}\ \bar{2}\ \bar{4}$ 5 3 4 2 0 1

8 8 7 6 ÷ 1 0 $\bar{1}\ \bar{2}\ \bar{4}$ 5 3/ 4 2 0 1

1 1 2 4   1 1 2 4

1 1 2 4

1 1 2 4

2 2 4/ 8

5 5/10 20

18/18 36 54

/34 34 68 136

---

1 1 1 2 $5_{1}8_{3}4/{}_{7}{}^{4}{}_{9}{}^{2}{}_{12}{}^{2}{}_{13}{}^{7}$

1 1 1 2 $6_{1}1$ 4/ 8 4 5 5 7

1 1 1 2 7 1 4/ 8 4 5 5 7

/ 8 8 16 32

---

1 1 1 2 7 1 $4/8/12_{1}{}^{3}{}_{2}{}^{1}{}_{3}{}^{9}$

1 1 1 2 7 1 4 18/1 3 5 4 9

1112772/1/3549
1124

1112723/$466_{1}3$ 1112723/4673

29.6.2 ILLUSTRATION 23:

Divide 146735679 by 888

First we change the Dividend into VINCULUM FORM as

8   1 5 $\bar{3}\ \bar{3}\ \bar{4}\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{1}$

8 8 8 ÷ 1 5 $\bar{3}\ \bar{3}$ 4 $\bar{4}$ / $\bar{3}\ \bar{2}\ \bar{1}$

1 1 2   1 1 2

6 6 12

4 4 8

9 9/ 18

29/ 29 58

/ 42 42 84

---

1 6 $4_{1}9_{2}9_{4}$ 2/ $8^{6}\ 9^{8}\ 8^{3}$

$1\ 6\ 4_1 1_1 3\ 2/8_1 5_1 6\ 3$

$1\ 6\ 5\ 2\ 3\ 2/9\ 6\ 6\ 3$

$1\ 6\ 5\ 2\ 3\ 2/9/6\ 6\ 3$

$9\ 9\ 18$

$1\ 6\ 5\ 2\ 3\ 2/9/15_1 5_2 1$

$1\ 6\ 5\ 2\ 3\ 2+9/1/6\ 7\ 1$

$1\ 1\ 2$

$1\ 6\ 5\ 2\ 4\ 1\ /1/\ 7\ 8\ 3$

$1\ 6\ 5\ 2\ 4\ 2\ /\ 7\ 8\ 3$

Q = 165242

R = 783

29.6.3 ILLUSTRATION 24

Divide 1 5 7 6 8 9 by 8 8 7

$1\ 5\ 7\ 6\ 8\ 9 = 2\ \bar{4}\ \bar{2}\ \bar{3}\ \bar{1}\ \bar{1}$

$8\ 8\ 7 \div 2\ \bar{4}\ \bar{2}/\bar{3}\ \bar{1}\ \bar{1}$

$2\ 2\ 6$

$\bar{2}\ \bar{2}\ \bar{6}$

$\bar{2}\ \bar{2}\ \bar{6}$

$\bar{2}\ \bar{2}\ \bar{2}/\bar{1}\ \bar{9}\ \bar{7}$

$1\ 7\ 8/\bar{1}/8\ 0\ 3$

$\bar{1}\ \bar{1}\ \bar{3}$

$1\ 7\ 8/1/7\ \bar{1}\ 0$

$1\ 7\ 8/\bar{1}/7\ \bar{1}\ 0$

$1\ 7\ 8-1/6\ 9\ 0$

177/690

29.6.4 ILLUSTRATION 25

Divide 13456786 by 87

$1\ 3\ 4\ 5\ 6\ 7\ 8\ 6 = 1\ 3\ 5\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{1}\ \bar{4}$

$$\begin{array}{l}
8\;7 \div 1\;3\;5\;\;\bar{4}\;\;\bar{3}\;\;\bar{2}/\;\bar{1}\;\;\bar{4}\\
\qquad\quad 1\;3\\
\qquad\qquad 4\;\;12\\
\qquad\qquad\quad 12\;\;36\\
\qquad\qquad\qquad\quad 20\;\;60\\
\qquad\qquad\qquad\qquad\quad 53\;\;159\\
\qquad\qquad\qquad\qquad\qquad\quad 111\;\;333\\
\hline
1\;\;4_{1}{}^{2}\;\;2^{0}\;\;5^{3}{}_{11}{}^{1}/269_{32}{}^{9}\\
1\;5\;4\;6\;4\;1/30/1\;\;9\\
\qquad\qquad\qquad\quad 3\;\;9\\
\qquad\qquad\qquad\qquad 3\;\;9\\
\hline
1\;5\;4\;6\;4\;1/33/13_{1}{}^{8}\\
1\;5\;4\;6\;4\;1\;+\;33/1/4\;\;8\\
\qquad\qquad\qquad\qquad\qquad 1\;\;3\\
\hline
1\;5\;4\;6\;7\;\;4/1/5_{1}{}^{1}\\
1\;5\;4\;6\;7\;\;4{+}1/61\\
154675/61\\
\qquad\qquad\quad Q = 154675\\
\qquad\qquad\quad R = 61
\end{array}$$

29.7 The NIKHILAM Method can further be extended even for computing the values in decimal places with equally ease and simplicity of procedure. There is almost nothing new in the procedure and methodology. The following illustrations shall be sufficient to explain this.

29.7.1 ILLUSTRATION 26

Divide 345675 by 9 upto 10 decimal places

$$\begin{array}{l}
9 \div 3\;\;4\;\;5\;\;6\;\;7\;/\;5\\
\qquad\quad 3\\
\qquad\qquad\; 7\\
\qquad\qquad\quad 12\\
\qquad\qquad\qquad\; 18\\
\qquad\qquad\qquad\qquad 25\\
\hline
3\;\;7\;\;1^{2}\;\;1^{8}\;\;2^{5}/\;\;30\\
3\;8\;4\;0\;5/3/0\\
\qquad\qquad\quad\;\; 3\\
\hline
3\;8\;4\;0\;5/3/3
\end{array}$$

38405+3/3

38408/3000000000/0

333333333/3

38408/3333333333/3

Hence Q = 38408.3333333333

29.7.2 ILLUSTRATION 27

Divide 2341 by 89 upto 3 decimal place

8 9 ÷ 2 3/4 1

111 2 2

5 5

2 5/1/16

11

2 5/1/27

26/2 7 0/ 0 0

2 2

2 9 9

9 11 11

$26/2\ 9_{1}1/20\ {}_{1}1$ = 26/301/2/11

22

26/301/2/33

= 26.303/33

So Q = 26.303

29.7.3 ILLUSTRATION 28

Divide 1234567890 by 888889 upto 7 decimal place

1234567890 = $124\overline{543211}0$

8 8 8 8 8 9 ÷ 1 2 4 $\bar{5}/\bar{4}\ \bar{3}\ \bar{2}\ \bar{1}\ \bar{1}$ 0

1 1 1 1 1 1 1 1 1/1 1 1

3 3/3 3 3 3

8/8 8 8 8 8

/7 7 7 7 7 7

$1\ 3\ 8\ 7/15_{1}6_{1}7_{1}7_{1}47$

1 3 8 7/1~~7~~678847

1111111

1 3 8 8/789958

We continue to devide

Remainder $7\ 8\ 9\ 9\ 5\ 8 = 1\ \bar{2}\ \bar{1}\ 0\ 0\ \bar{4}\ \bar{2}\ 5\ 0$

$$
\begin{array}{l}
1\ 3\ 8\ 8/1\ 2\ 1\ 0\ 0\ 4\ 2\ 0/\ 0\ 0\ 0\ 0\ 0\ 0 \\
\qquad\qquad 1\ 1\ 1\ 1\ 1\ 1\ 1 \\
\qquad\qquad\ \ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\bar{1}\bar{1} \\
\qquad\qquad\qquad \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \ \bar{1} \\
\qquad\qquad\qquad\ \ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \ \bar{1}\ \bar{1} \\
\qquad\qquad\qquad\qquad \bar{2}\ \bar{2}\ \bar{2}\ \ \bar{2}\ \bar{2}\ \bar{2}\ \bar{2} \\
\qquad\qquad\qquad\qquad\ \ \bar{8}\ \bar{8}\ \ \bar{8}\ \bar{8}\ \bar{8}\ \bar{8}\ \bar{8} \\
\qquad\qquad\qquad\qquad\quad \overline{14}\ \overline{14}\ \overline{14}\ \overline{14}\ \overline{14}\ \overline{14} \\
\qquad\qquad\qquad\qquad\qquad\quad \overline{27}\ \overline{27}\ \overline{27}\ \overline{27}\ \overline{27} \\
\hline
1\ 3\ 8\ 8/1\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{2}\ \bar{8}_{1}\bar{4}_{2}\bar{7}/5\bar{3}_{5}\bar{2}_{5}\bar{1}_{4}\bar{9}_{4}\bar{1}_{2}\bar{7} \\
1\ 3\ 8\ 8/1\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{2}\ \bar{9}\ \bar{6}\ \bar{7}/5\ \bar{8}\ \bar{7}\ \bar{6}\ \bar{3}\ \bar{3}\ \bar{7} \\
1\ 3\ 8\ 8/8\ 8\ 8\ 7\ 0\ 3\ 3/\bar{1}\ 4/1\ 2\ 3\ 6\ 6\ 3 \\
\qquad\qquad\qquad\qquad\qquad\qquad \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1}\ \bar{1} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad /3\ 3\ 3\ 3\ 3\ 3 \\
\hline
1\ 3\ 8\ 8/8\ 8\ 8\ 7\ 0\ 3\ 3/\bar{1}\ 3/3\ 4\ 5\ 8\ 8\ 7 \\
1\ 3\ 8\ 8/8\ 8\ 8\ 7\ 0\ 2\ 6/\ 3\ 4\ 5\ 8\ 8\ 6
\end{array}
$$

OR 1388.8887026/ 345886

Q = 1388.8887026

29.7.4 ILLUSTRATION 29

Divide 2467 by 8 upto 2 decimal place.

$$
\begin{array}{l}
8 \div\ 2\ 4\ 6/7 \\
2 \qquad\ \ 4 \\
\qquad\qquad\ 16 \\
\qquad\qquad\quad 44 \\
\hline
\qquad 2\ 8_{2}2/51 \\
\qquad 2_{1}0\ 2/51 \\
\qquad 5\ 0\ 2/5/11 \\
\qquad\qquad\qquad\ 10 \\
\hline
\qquad 3\ 0\ 2/5/1_{2}1 \\
\qquad 3\ 0\ 7/1/1 \\
\qquad\qquad\qquad\ 2 \\
\hline
\qquad 3\ 0\ 7/1/3
\end{array}
$$

We simplify and again continue

308/3 0 0

6

12

308/3 6_12

308/3 7 2

∴ Q = 308.37

29.7.5 __ILLUSTRATION 30__: Compute upto 3 places of decimal

98 ÷ 6 4 5 3 2

98 ÷ 1 $\bar{4}$ 4 5/3 2

02 0 2

0 $\bar{8}$

0/12

0 $\bar{6}$

1 $\bar{4}$ 6 $\bar{3}$/15 $\bar{4}$

6 5 7/1/46

02

∴ 6 5 7/1/48

Simplify and divide

658/ 4 8 0 0 0

0 8

016

016

658/ 4 8 8_16_16

4 8 9 7 6

Q = 658.489

CHAPTER 30

DIVISION- By Paravartaya Method

30.1 INTRODUCTION

We have seen how Nikhilam Method can be applied for division by real big digits but however this is not at all suitable for division by smaller number. Division by smaller number- and equally easy and superfast vedic method is available. This is based upon the PARAVARTAYA SUTRA. This is the fourth Sutra in the total list of sixteen and reads as .

It means transpose and apply. In other words we change the sign of the divisor digits and then apply the paravartaya method.

30.2 PROCEDURE FOR DIVISORS HAVING UNITY AS THEIR FIRST DIGIT

The procedure followed in such cases is exactly the same as we have seen in the case of division by Nikhilam method. The difference is only in initial steps, corresponding to the modification of divisor digits (MD). The stepwise process is illustrated below.

30.2.1 Illustration 1.

The following illustration shall explain the process.

Divide 1235 by 112 112 ÷ 1235

Step 1: We shall tabulate the divisor and dividend digits as follows

(divisor digits) 112 ÷ 1235 (dividend digits 4-numbers)
3-numbers

Step 2 i) As stated earlier, the divisor is such that its first digit is one, while modifying the divisor ignore this digit. Thus modified divisor (MD) will contain only two digits (here they are 1 2).

ii) Another modification in divisor is that all the remaining digits are taken down as digits of MD but with their sign reversed in every case. Thus here MD is $\overline{1}\overline{2}$. In other words we transpose and apply as per Paravartya Sutra number 4.

iii) Now MD contains two digits only, as such the oblique line shall be placed after two columns of dividend. As

$$\begin{array}{ccc} & \text{Columns} & \\ \text{3rd} & \text{2nd} & \text{1st} \\ 1 & 1 & 2 \\ (\text{M.D.}) & \bar{1} & \bar{2} \end{array} \qquad \begin{array}{cccc} \text{4th} & \text{3rd} & \text{2nd} & \text{1st} \\ 1 & 2\ / & 3 & 5 \end{array}$$

Step 3 All the remaining steps are the same as those already explained in the previous chapter of division by Nikhilam. Namely (i) complete the row of products obtained by multiplying with the first multiplier (appearing in the 4th column of D is directly our multiplier) by 2nd and 1st digit of MD and placing these below 3rd and 2nd columns of D respectively.

$$\begin{array}{ccc} 1 & 1 & 2 \\ & \bar{1} & \bar{2} \end{array} \quad \div \quad \begin{array}{cccc} 1 & 2\ / & 3 & 5 \\ & \bar{1} & \bar{2} & \\ \hline & 1 & & \\ \hline \end{array}$$

(ii) The second row will be obtained by finding an operator multipli (by summing the digits appearing the previous column of D here 3rd an multiplying the same with 2nd and 1st digits of MD and placing these below 2nd and 1st column of D. Here operator is 1($2+\bar{1}$ = 2-1 = 1)

$$\begin{array}{ccc} 1 & 1 & 2 \\ & \bar{1} & \bar{2} \end{array} \quad \div \quad \begin{array}{cccc} 1 & 2\ / & 3 & 5 \\ & \bar{1} & \bar{2} & \\ & & \bar{1} & \bar{2} \\ \hline 1 & 1\ / & & \end{array}$$

Here all the operations of multiplications have been exhausted. Since we have reached the first digit of D.

NOTE: This time since digits of MD are with negative sign the product may also have negative sign. Hence proper care of sign of product mus be taken in handling the vinculum numbers.

(iii) Draw oblique line just below the oblique line of D and algebric sum the digits column wise appearing on both sides of the line. The summation on R.H.S. of oblique line will be the Remainder R and the one on L.H.S. will be quotient Q.

$$\text{Here is} \quad 112 \ \div \quad \begin{array}{cccc} 1 & 2\ / & 3 & 5 \\ & \bar{1} & \bar{2} & \\ & & \bar{1} & \bar{2} \\ \hline 1 & 1\ / & 0 & 3 \end{array}$$

Result - R = 03
Q = 11

30.2.2 Illustration 2

Divide 13291 by 1125

Step 1 : We shall tabulate the divisor and dividend digits as follows -

1125 ÷ 13291

Step 2 : The divisor first digit is 1 so we ignore it. Further using Paravartya sutra our modified divisor is 125 with negative sign as $\bar{1}\ \bar{2}\ \bar{5}$.

(ii) Our MD contains three digits so the oblique line placed after three columns of D.

$$\begin{array}{rccccccc} & & \text{5th} & \text{4th} & & \text{3rd} & \text{2nd} & \text{1st} \\ 1125 & \div & 1 & 3 & / & 2 & 9 & 1 \\ \overline{125} & & & & & & & \end{array}$$

Step 3: The last digits of D is 1 this is our first multiplier. Multiply this with MD digits and place below the 4th, 3rd and 2nd digits

$$\begin{array}{cccc c cccccc} 1 & 1 & 2 & 5 & \div & 1 & 3 & / & 2 & 9 & 1 \\ & \bar{1} & \bar{2} & \bar{5} & & & \bar{1} & & \bar{2} & \bar{5} & \end{array}$$

Step 4: For next operator we add the digits of 4th column. So we get 2 as operator multiplier.

$$\begin{array}{cccc c cccccc} 1 & 1 & 2 & 5 & \div & 1 & 3 & / & 2 & 9 & 1 \\ & \bar{1} & \bar{2} & \bar{5} & & & \bar{1} & & \bar{2} & \bar{5} & \\ \hline & & & & & 1 & 2 & & & & \end{array}$$

Again multiply the operator digit 2 with MD digits, and placing at proper place we have reached the first digit. Hence this is our last step.

$$\begin{array}{cccc c cccccc} 1 & 1 & 2 & 5 & \div & 1 & 3 & / & 2 & 9 & 1 \\ & \bar{1} & \bar{2} & \bar{5} & & & \bar{1} & & \bar{2} & \bar{5} & \\ & & & & & & & & \bar{2} & \bar{4} & \overline{10} \\ \hline & & & & & 1 & 2 & / & & & \end{array}$$

Step 5: Now we add the digits of the last three columns to get the result

$$\begin{array}{cccc c cccccc} 1 & 1 & 2 & 5 & \div & 1 & 3 & / & 2 & 9 & 1 \\ & \bar{1} & \bar{2} & \bar{5} & & & \bar{1} & & \bar{2} & \bar{5} & \\ & & & & & & & & \bar{2} & \bar{4} & \overline{10} \\ \hline & & & & & 1 & 2 & / & \bar{2} & \bar{0} & \bar{9} \end{array} \text{ or } 11/1125 + \overline{209}$$

or 11/916

NOTE: Here we are left with the negative remainder (in vinculum fo as such we reduce the quotient by 1 and carry the full diviso to the remainder side (here 1125) and algebraically add to th negativeremainder to get the final R. This situation can arise quite often in division by Paravartya method.

2. However, it is interesting to observe that there is hardly an division being done in the whole process. Instead we just fol a series of simple digit by digit multiplications and algebra additions (with proper sign of course).

30.2.3 Illustration 3 :

Divide 12351 by 1012.

Step 1: We shall tabulate the divisor and dividend digits as before

1012 ÷ 12351

Step 2: (i)

$$\begin{array}{c} 1012 \\ \bar{0}\bar{1}\bar{2} \end{array} \div \quad 12351$$

(ii)

$$\begin{array}{c} 1012 \\ \bar{0}\bar{1}\bar{2} \end{array} \div \quad 12/351$$

Step 3: Multiplier is 1 here

$$\begin{array}{c} 1012 \\ \bar{0}\bar{1}\bar{2} \end{array} \div \quad \begin{array}{cccccc} 1 & 2 & / & 3 & 5 & 1 \\ & \bar{0} & & \bar{1} & \bar{2} & \\ \hline 1 & 2 & / & & & \end{array}$$

Step 4: Multiplier is 2 here

$$\begin{array}{cccc} 1 & 0 & 1 & 2 \\ & \bar{0} & \bar{1} & \bar{2} \end{array} \div \quad \begin{array}{cccccc} 1 & 2 & / & 3 & 5 & 1 \\ & \bar{0} & & \bar{1} & \bar{2} & \\ & & & \bar{0} & \bar{2} & \bar{4} \\ \hline 1 & 2 & / & 2 & 1 & \bar{3} \end{array}$$

Step 5: On normalising the result we get

Q = 1 2 / h = 207

30.2.4 Illustration 4:

Divide 1211 by 112

$$\begin{array}{lccc} & 1 & 1 & 2 \\ \text{M.D.} & & \bar{1} & \bar{2} \end{array} \quad : \quad \begin{array}{ccccc} 1 & 2 & / & 1 & 1 \\ & \bar{1} & & \bar{2} & \\ & & & \bar{1} & \bar{2} \\ \hline 1 & 1 & / & \bar{2} & \bar{1} \end{array}$$

Q = 11 R = $\bar{2}\bar{1}$

Once again here the remainder is all negative, which is not accept Hence we reduce quotient by one and add 112 algebrically to the remainder. The modified quotient is 10 (11-1) and remainder is

$112 + \overline{21} = 112 - 21 = 91$

30.2.5 Illustration 5 :

Divide 13254 by 1132

$$\begin{array}{ccccccccccc}
1 & 1 & 3 & 2 & \div & 1 & 3 & / & 2 & 5 & 4 \\
 & \bar{1} & \bar{3} & \bar{2} & & & \bar{1} & & \bar{3} & \bar{2} & \\
 & & & & & & & & \bar{2} & \bar{6} & \bar{4} \\
\hline
 & & & & & 1 & 2 & / & \bar{3} & \bar{3} & \bar{0}
\end{array}$$

Normalise as before

or 11/802 (R = 1132-330)

30.3 DIVISION BY BIG NUMBERS

Now let us once again take up the division by big numbers and try to perform the same by the paravartaya method. But as as first additional step we shall convert the big divisor digit to the vinculum form and then follow the same step by step method. The following examples shall illustrate the details.

30.3.1 Illustration 6 :

Divide 112 by 89

Here by using Vinculum divisor can be modified as $89 = 1\bar{1}\bar{1}$.

Now by paravartaya sutra transpose of last two digits is taken

$$\begin{array}{ccccccc}
1 & \bar{1} & \bar{1} & \div & 1 & / & 1\ 2 \\
 & 1 & 1 & & & & 1\ 1 \\
\hline
 & & & & 1 & / & 2\ 3
\end{array}$$

Note: We have got the same MD, as in case of Nikhilam method.

30.3.2 Illustration 7:

Divide 40000 by 9808

Here $9808 = 1\ 0\ \bar{2}\ 1\ \bar{2}$

$$\begin{array}{ccccccccccc}
1 & 0 & \bar{2} & 1 & \bar{2} & \div & 4 & / & 0 & 0 & 0\ 0 \\
 & 0 & 2 & \bar{1} & 2 & & & & 0 & 8 & \bar{4}\ 8 \\
\hline
 & & & & & & 4 & / & 0 & 8 & \bar{4}\ 8
\end{array}$$

, Normalising

OR 4/0768

30.3.3 Illustration 8:

Divide 1112 by 849 $\qquad 849 = 1\ \bar{2}\ 5\ \bar{1}$

$$\begin{array}{cccccccccc}
1 & \bar{2} & 5 & \bar{1} & \div & 1 & / & 1 & 1 & 2 \\
 & 2 & \bar{5} & 1 & & & & 2 & \bar{5} & 1 \\
\hline
 & & & & & \bar{1} & / & 3 & \bar{4} & 3
\end{array}$$

OR 1 / 2 6 3

30.3.4 Illustration 9:

Divide 81946 by 898

Here both divisor and dividend need to be modified by Vinculum approach.

$$\begin{array}{lcl}
898 & = & 1\overline{1}0\overline{2} \\
81946 & = & 1\overline{2}2\overline{1}5\overline{4} \\
1\overline{1}0\overline{2} & \div & 1\overline{2}2 / \overline{1}5\overline{4} \\
102 & & 10 \, 2 \\
 & & \overline{1} \, 0\overline{2} \\
 & & \, 102 \\
\hline
 & & 1\overline{1}1 / 23\overline{2}
\end{array}$$

or 91 / 228

Note: Such a difficult problem|how easy it has become in the Vedic approach. All the big digits have gone. The whole operation is a smiling job.

30.3.5 Illustration 10 :

Divide 2672 by 838

$$\begin{array}{lcl}
838 & & \\
1\overline{2}4\overline{2} & \div & 2672 \\
2\overline{4}2 & & 3/\overline{3}\,\overline{3}\,2 \\
 & & 6\overline{12}\,6 \\
\hline
 & & 3/3\overline{1}\overline{5}\;8 \quad \text{or} \quad 3/2\overline{5}8 \\
 & & \text{or } 3/158
\end{array}$$

30.4 DISCUSSIONS

In the above examples we have seen how much of similarity in the NIKHILAM and the PARAVARTAYA METHOD. As a matter of fact if we critically observe we have got exactly the same modified divisor even by using the paravartaya sutra.

2. Further the operational ease and simplicity of the magic vedic method continous to exit in both the nikhilam as well as the paravartaya method.

30.5 PROCEDURE FOR OTHER DIVISORS

All the previous examples had first digit as a unity in the divisor . But if this condition is not satisfied, we just need a small additional step as explained in the following examples. like the multiplication we use the Anurupena Sub-sutra in these cases.

30.5.1 Illustration 11

Divide 1354 by 184

Here 184 can be written as $2\bar{2}4$. In order to apply paravartaya Sutra (first digit must be unity) . For this we divide this divisor by 2. Thus the new divisor becomes $1\bar{1}2$.

$$\begin{array}{rcl} 1\,8\,4 & \div & 1\,3\,/\,5\,4 \\ 2/\,2\,\bar{2}\,4 & \div & 1\;\;\bar{2} \\ 1\,\bar{1}\,2 & & \quad 4\,\bar{8} \\ 1\,\bar{2} & & \\ & & \overline{2/\,1\,4\,/\,7\,0\,\bar{4}} \end{array}$$

Note: The further operations are just the same without any change

Here since divisor has been divided by 2, as such the raw quotient is also divided by 2 in order to get the final quotient. However, nothing is done to remainder, which remains unchanged. In this process we have again used the same upsutra Anurupena which was used in the subbase cases of multiplication.

30.5.2 Illustration 12

Divide 1233 by 224

Here we sub divide divisor by 2 and then proceed

$$\begin{array}{rcl} 2\,/\,224 & \div & 1\,2\,/\,3\,3 \\ 112 & & \bar{1}\;\;\bar{2} \\ \bar{1}\bar{2} & & \quad \bar{1}\,\bar{2} \\ & & \overline{2/\,1\,1\,/\,0\,1} \\ & & 5\,/\,113 \end{array}$$

when the quotient (i.e. 11) is divided by sub diviser (i.e. 2) it yields five and half. This means quotient is 5 and half of original divisor (i.e. half of 224 = 112) is to be transferred towards remainder side. Thus the remainder becomes 112 + 1 = 113.

30.5.3 Illustration 13

Divide 7684 by 672

Here if we divide divisor by 6 we get 112. Hence

$$\begin{array}{rcl} 6\,/\,672 & \div & 1\,\bar{2}\,\bar{3}\,/\,\bar{2}\,4 \\ 112 & & \bar{1}\,\bar{2} \\ \bar{1}\bar{2} & & \quad 3\quad 6 \\ & & \qquad 2\,4 \\ & & \overline{1\,\bar{3}\,\bar{2}\,/\,6\,8} \end{array} \quad \text{or} \quad \underline{6}/\,68/68 \quad \text{or} \quad 11\tfrac{1}{3}/68$$

As $\frac{1}{3}$ of 672 = 224

and R = 224 + 68 = 292

Hence 11/292. is the result.

Alternatively, we could also proceed by multiplying divisor by 2. Thus

$2 \times 672 \div 1\bar{2} / \bar{3}\ \bar{2}\ 4$

$1344 \qquad \bar{3} \quad \bar{4}\ \bar{4}$

$\bar{3}\bar{4}\bar{4} \qquad 1\bar{5}\ 20\ 20$

$1\ \bar{5} / 10\ \bar{4}\ 4$

or $5 / 964$

By Anurupena Q = $5 \times 2 = 10$

Remainder (= 964) is greater than divisor (i.e. 672) Hence corrected quotient and remainder will be 11 and 292 (R = 964 - 672 = 292 and Q = 10+1 = 11) respectively.

30.6 DISCUSSIONS

1. By judicious use of vinculum, submultiplies or subdivisor as the case may be, we find through examples given above, that all the problems of divisors can be easily performed using paravartaya sutra. Hence it is versatile sutra for performing division.

2. Most of the examples given above establish that divisions by Nikhilam and by paravartaya are equivalent.

3. Divisions can be performed in number of other ways but most suitable ones are illustrated above.

30.7 The following examples shall further elaborate the Vedic paravartaya method of division.

30.7.1 Illustration 14

Divide 45678 by 198

The dividend and divisor can be written as

$45678 = 5\bar{4}\bar{3}\bar{2}\bar{2}$

$198 = 20\bar{2}$

$20\bar{2} \div 5\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{2}$

Using **PARAVARTAYA SUTRA** the first digit of divisor must be changed into unity, i.e. $20\bar{2}$ is divided by 2 we get $10\bar{1}$.

$2 / 20\bar{2} \quad \div \quad 5\ \bar{4}\ \bar{3} / \bar{2}\ \bar{2}$

$\quad 10\bar{1} \quad\quad 0\ 5$

$\quad\quad\quad\quad\quad 0\ \ \bar{4}$

$\quad 01 \quad\quad\quad\quad 0\ 2$

$\quad\quad\quad 5\ \bar{4}\ 2 / \bar{6}\ 0$

Using Anurupena we modify the quotient.

$2 / 462/\bar{1}40$

$\quad 231/\bar{1}/40$

$\quad\quad\quad 01$

$231/\ \bar{1}/\ 41$

231-1/39

230/39

Q = 230 R = 39

30.7.2 Illustration 15

Divide 165789 by 111

$1\ 1\ 1 \quad \div \quad 1\ 6\ 5\ 7\ 8\ 9$

$\bar{1}\ \bar{1} \quad \div \quad 2\ \bar{3}\ \bar{4}\ \bar{2} / \bar{1}\ \bar{1}$

$\quad\quad \bar{2}\ \bar{2}$

$\quad\quad\quad 5\ 5$

$\quad\quad\quad\quad 1 / 1\ 1$

$\quad\quad\quad\quad\quad 4\ \bar{4}$

$2\ \bar{5}\ \bar{1}\ \bar{4} / \bar{4}\ \bar{5}$

$1\ 4\ 9\ 4/\ \bar{1} / 55$

$\quad\quad\quad\quad\quad\quad 11$

$1494/\ \bar{1} / 66$

$1494+\bar{1} / 66$

1493/66

Q = 1493 R = 66

30.7.3 Illustration 16

Divide 16542305 by 9889

$16542305 = 2\bar{4}542305$

$9889 = 10\bar{1}\bar{1}\bar{1}$

$10\bar{1}\bar{1}\bar{1} \quad \div \quad 2\bar{4}54 / 2305$

$0111 \quad\quad 022 / 2$

$\quad\quad\quad \bar{4} / \bar{4}\ \bar{4}$

$\quad\quad\quad 0 / 7\ 7\ 7$

$\quad\quad\quad\quad / 0\ 2\ 2\ 2$

$2\bar{4}72/ 7897$

1672/ 7897

Therefore Q = 1672 R = 7897

30.8. DECIMAL FORM

The Paravartya method can also be extended for getting the answer in decimal form with equal ease and simplicity.

30.8.1 Illustration 17

Divide 563478 by 112 upto 3 decimal places

$$\begin{array}{lll}
112 & \div & 563478 \\
112 & \div & 1\ \bar{4}\ \bar{4}\ 3\ 5\ /\ \bar{2}\ \bar{2} \\
\overline{12} & & \quad \bar{1}\ \bar{2} \\
 & & \qquad 5\ 10 \\
 & & \qquad\quad 1\ 2 \\
 & & \qquad\qquad \overline{14}\ \overline{28} \\
 & & \qquad\qquad\quad 7\ 14 \\
\hline
 & & 1\bar{5}\ \bar{1}\ 14\bar{7}/\ 2\bar{3}\ 12
\end{array}$$

$$\begin{array}{l}
1\ \bar{5}04\bar{7}/\bar{2}\bar{2}2 \\
5033/\bar{1}7/82 \\
\qquad\quad 1\ 2 \\
\qquad\qquad \bar{8}\ \overline{16} \\
\hline
5033/\bar{1}8/2_{\bar{1}}\bar{4} \\
5033+\ \bar{1}8/1\bar{4} \\
5031/6
\end{array}$$

We continue further

$$\begin{array}{l}
5\ 0\ 3\ 1\ /\ 6\ 0\ 0\ /\ 0 \\
\qquad\qquad\quad \bar{6}\ \overline{12} \\
\qquad\qquad\qquad 6\ \ 12 \\
\hline
5\ 0\ 3\ 1\ /\ 6\ \bar{6}\ \bar{6}\ /\ 1\ 2 \\
5\ 0\ 3\ 1\ /\ 5\ 3\ 4\ /\ 1\ 2 \\
5\ 0\ 3\ 1.\ 5\ 3\ 4\ /\ 1\ 2
\end{array}$$

Therefore Q = 5031.534

etc.

CHAPTER 31

ALGEBRAIC DIVISION

31.1 INTRODUCTION: In this method of division we use the same Vedic formulae Paravartya Yojayet which means transpose and apply. It is interesting to observe that the vedic method of division for algebraic equations is exactly similar to that for the numerical division as already explained in the previous chapter. As a matter of fact in vedic method it seems difficult to draw a line where arithmetic stops and algebra starts. The following examples shall bring out the points of similarity and differences.

31.2 TYPE 1:- UNITY COEFFICIENT

We first of all take the division by algebraic expressions having unity coefficient of the first term of the divisor.

31.2.1 ILLUSTRATION 1: Divide $9x^2 - 12x - 42$ by $x-1$.

Step 1 : We shall tabulate the divisor and dividend digits as follows

$$\underline{x-1} \,/\, \overline{9x^2 - 12x - 42}$$

Step 2 : The coefficient of the first term of the Divisor is 1. So ignored it and modifying the rest of the divisor and change its sign. Here our modified divisor is 1.

$$\begin{array}{l} \underline{x-1} \,/\, \overline{9x^2 - 12x - 42} \\ \quad 1 \end{array}$$

(ii) Draw a oblique line just before the last term of the dividend. As the highest power of divisor is x. We shall have only constant term as the remainder.

$$\begin{array}{l} \underline{x-1} \,/\, 9x^2 - 12x / - 42 \\ \quad 1 \end{array}$$

Step 3 : The remaining steps are the same as described in the previous chapter. We carry down the coefficient of the first term without any change (Here 9). Now we multiply the operator 1 with the 1^{st} term of dividend and place the product just below the 2^{nd} term

$$\begin{array}{l|lll} \underline{x-1} & 9x^2 & -12x & /-42 \\ 1 & & 9 & \\ \hline & 9 & & \end{array}$$

(ii) For the next operator we add the terms of the 2^{nd} column

$$\begin{array}{l|lll} \underline{x-1} & 9x^2 & -12x & /-42 \\ 1 & & 9 & \\ \hline & 9 & -3 & \end{array}$$

Our next operator is 1- 3.

Step 4: Again we place the product of the operator and the MD digit just below the third term

$$\begin{array}{l|lll} \underline{x-1} & 9x^2 & -12x & /-42 \\ 1 & & 9 & -3 \\ \hline & 9 & -3 & \end{array}$$

(ii) We add the 3^{rd} column and so on. We go on repeating the steps till we reach the last term

$$\begin{array}{l|lll} \underline{x-1} & 9x^2 & -12x & /-42 \\ & & +9 & -3 \\ \hline & 9 & -3 & /-45 \end{array}$$

Step 5 : Now the numbers obtained in each column are the coefficients of the respective terms with appropriate sign, both in the quotient and the remainder. The powers of x in the quotient are obtained by reducing the powers of the dividend terms by the power of the highest term in the divisor. In this example divisor has only x therefore we reduce the powers by 1.

$$\begin{array}{l|lll} \underline{x-1} & 9x^2 & -12x & /-42 \\ & & 9 & -3 \\ \hline & 9x & -3 & /-45 \end{array}$$

So $Q = 9x - 3$

$R = -45$

31.2.2 Illustration 2: Divide $5x^2 + 20x + 14$ by $x - 2$

Step 1 : Tabulate the divisor and the dividend expressions as follows.

$$\underline{x-2}\,/\,\overline{5x^2 + 20x + 14}$$

Step 2 : Thus the modified divisor is 2 which is our 1st operator.

$$\begin{array}{l l} \underline{x-2}\,/ & \overline{5x^2 + 20x + 14} \\ \ \ 2 & \end{array}$$

(ii) Draw oblique line before the constant term 14 of the dividend.

$$\begin{array}{l l} \underline{x-2}\,/ & \overline{5x^2 + 20x/+ 14} \\ \ \ 2 & \end{array}$$

Step 3: We carry down the coefficient of the first term, here 5. Place the product of the operator and the first coefficient of dividend expression below the 2nd column. We get 2x5 = 10.

$$\begin{array}{l c c c} \underline{x-2}\,/ & 5x^2 & +\ 20x/ & +14 \\ \ \ 2 & & 10 & \\ \hline & 5 & +\ 30/ & \end{array} \qquad \text{Operator is } 30$$

Step 4: (i) Again multiply the operator with divisor operator and place below the 3rd column. Here 30 x 2 = 60.

$$\begin{array}{l c c c} \underline{x-2}\,/ & 5x^2 & +\ 20x/ & +14 \\ \ \ 2 & & 10 & 60 \\ \hline & 5 & 30 & \end{array}$$

(ii) Add the terms of 3rd column

$$\begin{array}{l c c c} \underline{x-2}\,/ & 5x^2 & +\ 20x/ & +\ 14 \\ & & 10 & 60 \\ \hline & 5 & +\ 30/ & 74 \end{array}$$

Step 5: Once again the power of x are obtained by reducing the power of the dividend term by the highest power of the divisor.(Here 1).

$$\begin{array}{r|rrr}
\underline{x-2} & 5x^2 & +20x & -114 \\
 & & 10 & 60 \\ \hline
 & 5x & +30/ & 74
\end{array}$$

$Q = 5x+ 30 \quad P = 74$

31.2.3 ILLUSTRATION 3 + Divide $4x^2 + 15 x - 13$ by $x+1$

$$\begin{array}{r|rrr}
\underline{x+1} & 4x^2 & +15x/ & -13 \\
-1 & & -4 & -11 \\ \hline
 & 4x & +11/ & -24
\end{array}$$

$Q = 4 x+11 \quad R = - 24$

31.2.4 ILLUSTRATION 4 : Divide $x^3 + 5x^2 - 20x + 14$ by $x-3$

$$\begin{array}{r|rrrr}
\underline{x-3} & x^3 & +5x^2 & -20x/ & +14 \\
3 & & 3 & 24 & 12 \\ \hline
 & x^2 & +8x & +4/ & 26
\end{array}$$

$Q = x^2 + 8x + 4 \quad R = 26$

31.2.5 ILLUSTRATION 5: Divide $x^3 - 3x + 15$ by $x+2$

$$\begin{array}{r|rrrr}
\underline{x+2} & x^3 & +0x^2 & -3x/ & +15 \\
-2 & & -2 & 4 & -2 \\ \hline
 & x^2 & -2x & +1 / & 13
\end{array}$$

$Q = x^2 - 2x +1 \quad R = 13$

Note:- Since the expression of dividend does not contain ter for x^2 hence it is essential to place a term $0x^2$ and then carryout the division as usual.

31.2.6 ILLUSTRATION 6 + Divide $x^4-x^3+x^2-3x+4$ by x^2-x+1. Here we operate the two coefficients at each stage of division.

$\underline{x^2-x+1} / \; x^4-x^3+x^2/-3x+4$

$1 \; -1 \qquad 1 \; -1$

$\qquad\qquad 0 \quad 0$

$\qquad\qquad\qquad 0 \quad 0$

$|x^2+0x+0/ -3x+4$

$Q = x^2 \qquad R = -3x+4$

Notes: Since the divisor has the x^2 term we can therefore get the remainder with x terms, as such we draw the oblique line, demarcating the remainder portion before the x term of the dividend.

2. Please note that we do not modify the power of x in the remainder terms.

3. We also write even the row consisting of all zero terms, to avoid error.

31.2.7 ILLUSTRATION 7 : Divide $5x^4+7x^3-3x^2+15x+24$ by x^2-2x+3

$\underline{x^2-2x+3}/5x^4+7x^3-3x^2/+15x+24$

$2 \; -3 \qquad 10 \; -15$

$\qquad\qquad 34 \; -51$

$\qquad\qquad\qquad 32 \; -48$

$5 \; +17 \; +16/ -4 \; -24$

OR $5x^2+17x+16/ -4x-24$

$Q = 5x^2+17x+16 \qquad R = -4x-24$

31.2.8 ILLUSTRATION 8: Divide $3x^4-2x^3-3x-2$ by x^2+1

$\underline{x^2-0x+1}/ \; 3x^4-2x^3+0x^2/-3x-2$

$0 \; -1 \qquad 0 \; -3$

$\qquad\qquad 0 \quad 2$

$\qquad\qquad 0 \quad +0 \; +3$

$3 \; -2 \; -3 / -1 \; +1$

$Q = 3x^2-2x-3 \qquad R = -x+2$

OR $3x^2-2x-3/ -x+1$

Note: The terms of $0x^2$ and $0x$ in dividend and divisor respectively are added before division.

31.3 TYPE II : In all the above cases, the first coefficient of Divisor terms was unity. But in these cases where this coefficient is other than unity ~~than~~ the whole divisor should be divided by a suitable number such that its coefficient becomes 1. The quotient so obtained is also divided by the same number. However the Remainder will be unchanged. However the remaining process is the same as above. Now let us take some examples of this type. Please note that we are essentially using the Anurupena upstra here.

31.3.1 ILLUSTRATION 9 : Divide $2x^3-5x^2+8x-10$ by $2x-4$

$$\begin{array}{llrrrr} & 2)\underline{2x-4}/ & 2x^3 & -5x^2 & +8x/ & -10 \\ \text{OR} & x-2 & & 4 & -2 & 12 \\ \hline \text{M.D.}\,{-2} & 2) & 2x^2 & -1x & +6/ & 2 \end{array}$$

$$x^2 - \frac{1}{2}x + 3\ /2 \quad \text{By Anurupena}$$

$$Q = x^2 - 1/2\,x + 3 \qquad R = 2$$

The remainder is not modified

31.3.2 ILLUSTRATION 10 + Divide $4x^5-8x^4+6x^3+12x^2-16x+20$ by $2x^2-2x+1$

$$\begin{array}{llrrrrrr} & 2)\underline{2x^2-2x+1}/ & 4x^5 & -8x^4 & +6x^3 & +12x^2/ & -16x & +20 \\ & \underline{x^2-x+\frac{1}{2}} & & 4 & -2 & & & \\ \text{M.D} \rightarrow & 1\ -\frac{1}{2} & & & -4 & 2 & & \\ & & & & & 0 & 0 & \\ & & & & & & 14 & -7 \\ \hline & & 4 & -4 & +0 & +14/ & -2 & +13 \end{array}$$

$$\text{OR } 2)4x^3-4x^2+0\,x+14/\ -2x+13$$

Hence $Q = 2x^3 - 2x^2 + 7$

$R = -2x + 13$

(7) In the case of division of the algebraic expressions, we can also use the converse of the URDHAVA TIRYAK SUTRA this results in simple argumentation process which is quite similar to the paravartaya method, in essence. However by argumentation we can straight away write down the quotient and the remainder in one line The detailed discussion of this with a number of solved examples has been presented in a seperate chapter. and further details are available in Swami ji's book.

31.3.3 ILLUSTRATION 11: Divide $5x^2 - 6x + 5$ by $3x-9$

$$\begin{array}{lll} & 3)\underline{3x-9} & /5x^2 - 6x + 5 \\ & \underline{x\ -3} & \underline{\quad 15 \quad\ 27} \\ \text{MD} \rightarrow & 3 & \\ & & 3)\ 5x + 9/32 \\ & & \frac{5}{3}x + 3\ /32 \end{array}$$

$Q = \frac{5}{3}x + 3 \qquad R = 32$

31.4 DISCUSSIONS

(1) The above examples explain the Superfast and easy vedic methods of division by using the paravartaya Sutra.

(2) In these three chapters we have seen that the Vedic methods of division do not require the multiplication of large number divisor by the trial digits at every step, as required in the present method.

(3) The Vedic methods are rather easy and the chances of error in working with them are drastically reduced.

(4) It is interesting to point out that the so called Remainder theorem, which is specially taught in the algebra, gives only the remainder of an algebraic expressions. However the paravartaya sutra goes much ahead and yields both the quotient and the remainder too.

(5) The Vedic methods go further ahead in providing us with the unique simple formula for checking the correctness of the results of our division both for arithmetic and algebraic problems.

(6) However the methods discussed so far are suitable for special cases and the general DHUAJANKA-METHOD is applicable for all cases without exception.

CHAPTER - 32

STRAIGHT DIVISION

32.1 INTRODUCTION:

There is yet another method of division in vedic style. This involves a bit more of mental working but the answer comes straight away in one step. The method is evolved by the combination of two vedic sutras, namely Urdhav Tiryak and Dhvajanka (meaning on top of the flag).

32.2 Let us take some examples in order to illustrate the new Dhvajanka method.

32.2.1 ILLUSTRATION 1+ Divide 46956 by 65

65 ÷ 46956

Step 1: Out of the divisor 65, we put down first digit(ie6) in the diviser column as operator and put other digit (ie5) on top of the flag as Flag digit as shown below:

Flag Digit
5 ÷ 46956
(operator)6

The entire division is to be done by 6.

Step 2 As one digit has been put on the top we allot one place (at the right end of D) to the remainder portion of the answer and mark off from the other digits by vertical dots - Here last digit 6 has been marked off

5:4695:6:
6

Step 3: (i) We divide 46 (of dividend) by 6 and get 7 as the quotient and 4 as remainder, we put down 7 as the first quotient digit and prefix the remainder (4) up before the next digit (i.e. 9) of the dividend. Then our actual gross dividend

(G.D) is 49 for next step.

6^{5} :4695 :6:
4
7

(ii) from 49 we subtract the product of the flag digit (i.e.5) and the first quotient digit (i.e. 7). So we get 49 -
7x5 = 35
14

Thus 14 is our net divident for further operation.

Step 4: (i) Now divide this net divided i.e. 14 by 6 we get 2 as second quotient digit and 2 as remainder which is placed before the next digit of divident (i.e.5). Now our Gross dividend is 25.

65 : 4695:6:
6 42
72

(ii) Again from 25 we substract the product of the flag digit (5) and the 2^{nd} quotient digit (i.e.2). So we get 25 - (2x5) = 25 - 10 = 15. This is the next net dividend.

Step 5:- (i) On dividing 15 by 6 we get 2 as third quotient digit and 3 as remainder to be placed before 6. Now 36 is out next gross dividend

5 : 4695:6:
6 423
722

(ii) Out of 36 we subtract the product of the flag digit (5) and the 3^{rd} quotient digit (i.e.2). So we get 36 - (2x5) =36-1
= 26.

Hence 26 is the remainder.

5:4695:6:
6 423
722:26:

Therefore Q = 722

R = 26

32.2.2 ILLUSTRATION 2: Divide 518 by 23

23 ÷ 518

Step 1: We write it as follows

F.D.
3 ÷ 518
(operator)2
Here 3 is the flag digit

Step2: We put dots for the remainder portion

3 : 51 :8:
2

Step 3: (i) Divide 5 by 2 we get 2 as first quotient digit [illegible] the remainder and place it before the next digit as

3 : 51 :8:
2 1
2

So our next Gross divided is 11 , 11 ND = 11 - 3x2 = 5

Step 4: Divide net divident(i.e.5) by 2, we get 2 as the 2nd quotient digit and 1 as remainder and place it before 8 as

3 : 51 :8:
2 11
22

Next gross dividend is 18.

(ii) From 18 subtract

(3x2)=-6
12

So 12 is our remainder

 3 : 51 : 8
2   11
 22 :12

Q = 22
R = 12

The whole process of step by step division as explained in the previous two examples can be done mentally and there is absolutely no need to write down the steps. By using this Vedic method of direct division we can get the total answer in one line using the simple mental working. Let us try to understand the following illustration by mentally doing the mental working as per the previous steps.

32.2.3 ILLUSTRATION 3: Divide 5120 by 64

 4 : 512 : 0 :
6
   3
 80 : 0 :

The second net divident is 0 [32 - 8x4]

32.2.4 ILLUSTRATION 4: Divide 7648 by 94

 4 :764 :8:
   43
 81 :34:

32.3 In the previous examples we had seen that out of the two digits of divisor we are actually dividing by using two digits and the flag digit is being used for simple readjust of the gross dividend. Now let us take few examples of divis by the three digit numbers.

32.3.1 ILLUSTRATION 5 Divide 7142695 by 824.

Here the divisor is of 3 digit. In this case we place last two digits of divisor as flag digits and adopt a slightly different modus operadi on Urdhva Tiryak lines as follows

$$\begin{array}{l} 24 : 71426 : 95 : \\ 8 \end{array}$$

The initial two steps are done as already explained.

Step 1 (i) Divide 71 by 8 we get 8 and remainder is 7.

$$\begin{array}{rl} 24: & 71426 :95 \\ 8 & 7 \\ \hline & 8 \end{array}$$

So 74 is the Gross dividend.

(ii) From 74 we subtract the product of the 1st digit of flag digit (i.e. 2) and the 1st quotient digit (i.e. 8). So we get 74 - (8x2) = 74 - 16 = 58. So 58 is our net next dividend.

The subsequent steps have a slight modifications. We use the Urdhva Sutra for adjustment of our gross dividend.

Step 2:(i) Divide 58 by 8 we get 6 and 10 as remainder. So our next gross dividend is 102.

$$\begin{array}{rl} 24 : & 71426 : 95 : \\ & 710 \\ \hline & 86 \end{array}$$

(ii) From 102 we subtract,(by Urdhva tirvak rule) gross products of two flag digits (24) and two quotient digits(86)i.e.

$$\begin{array}{ll} 24 & \rightarrow 12 + 32 \\ \text{x} & \\ 86 & \qquad \rightarrow 44 \end{array}$$

We get remainder is

$$\begin{array}{r} 102 \\ -\ 44 \\ \hline 58 \end{array}$$ (which is our next dividend).

Step 9: (i) On dividing 58 by 8 we get 6 and 10 as remainder.

Our next gross dividend is 106.

24 : 71426: 95 :
8
7 10 10
8 6 6

(ii) From 106 we subtract, by Urdhav Tiryak rule gross products of two flag digit (24) and the last 2 quotient digits (66) i.e.

24
X → 12 + 24 → 36
66

and the remainder is 106
−36
70 which is our next net dividend.

Step 5: (i) Dividing 70 by 8 we get 8 and 6 as remainder. Out next gross dividend is 69

24 : 71426 : 9 5 :
8
7 10 10 6
8 6 6 8

(ii) From 69 we deduct by Urdhav tiryak rule, gross product of two flag digits(24) and last 2 quotient digit 68 i.e.

24
x → 16 + 24
68 → 40

the remainder is 69
− 40
29 (which is our net dividend) and 295 is therefore the gross R.

32.3.2 ILLUSTRATION 6 Divide 16385 by 128

128 ÷ 16385

In this example we observe that it is more convenient to work with 1 flag digit (8 here).

Step 1 : 8 ÷ 16385
12

Step 2: 8 : 1638 :5:
12

We make 8 as the flag digit

Step 3:(i) 8 : 1638 :5: Divide 16 by 12
12 4
1

(ii) From 43(G.D)
- 8=(8x1)
35 → next net dividend

Step 4: Divide 35 by 12 we get 2 and remainder 11.

8 : 1638:5:
4 11
12

(ii) From 118(G.D)
- 16(2x8)
102 → Next net dividend

Step 5: Divide 102 by 12 we get 8 and remainder is 6.

8 : 1638 :5:
12 4 11 6
128

(ii) From 65 (G.D)
64 (8x8)
1 Remainder

Hence 8 : 1638 :5:
12 4 11 6
128 :1:

32.3.3 ILLUSTRATION 7 Divide 24829685 by 2365

Step 1: The divisor can be written as

65 : 2 4 8 2 9 6 : 8 5 :

23

Here 23 (operator) and 65 (Flag Digits)

The entire division is to be done by 23

Step 2:- Divide 24 by 23, we get quotient digit (i.e.1) and the remainder is (i.e.1). The remainder is placed before the next digit (i.e.8) and the quotient is placed below the line

```
        6 5 : 2 4 8 2 9 6 : 8 5 :
   2 3          1
             ____________________
             1
```

Our next gross dividend is 18.

(ii) Subtract the product of the quotient digit and the first flag digit from 18(i.e. 18 - 6 = 12) which is the net dividend at this stage.

Step 3: Divide the net dividend 12 by 23 which is smaller than the divisor. So 12 is placed before the next digit (2) as our remainder (of this step) and the quotient is 0.

```
        6 5 : 2 4 8 2 9 6 : 8 5
   2 3            1 122
               __________________
                  1 0
```

So our next gross dividend is 122.

(ii) Now subtract the product of the 2 quotient digits (i.e.1
and the 2 flag digits (i.e. 65) by the Urdhava Tiryak Sutra
65 x10 = 5 (by cross product)5, from 122 we get (122 - 5 = 117) which
is the next net dividend.

Step 4: Again divide the net dividend (117) by 23 we get quotient digit (i.e.4) and the remainder (25) place it before the next digit (i.e.9)

```
          6 5 : 2 4 8 2 9 6 : 8 5
   2 3              1 12 25
                 __________________
                    1 0 4
                 __________________
```

So our next g.d. is 259.

(ii) Again subtract the product of 65 (the flag digits) and the last two digits of quotient from 259 (i.e. 259 - (65 x 04 = 24) = 235)., the cross product $\frac{65}{04} = 24$
So 235 is our net dividend.

Step 5: (i) Further divide 235 by 23, we get 9 as quotient digit and the remainder 28 is placed before the next digit (i.e. 6).

```
         6 5 : 2 4 8 2 9 6 : 8 5
   2 3           1 12 25 28
           ________________________
           1 0 4 9
```

Therefore 286 is our next gross dividend. See 32.4

(ii) Further subtract the product of the flag digit (65) and the last two digits of the quotient (i.e. 49) from 286 we get

286 - (65 x 49 = 74) = 212 (net dividend)

Step 6:- (i) Divide the net dividend 212 by 23 we get quotient digit 8 and the remainder 28, place it before the next digit (i.e.8)

```
            6 5 : 2 4 8 2 9 6 : 8 5
      2 3
                      1 12 25 28 28
                 ______________________
                      1 0 4 9 8 :
```

Hence 288 is next gross dividend.

(ii) Subtract the product of flag digit (65) and the last two digits of the quotient (i.e. 98) from 288. We get (288 - (65 x 98 = 93) = 105) 195 which is placed before the next digit 57 . So the gross R is 1955.

Step 7:- To find the net remainder again subtract the product of the last flag digit (i.e.5) and the last quotient digit (i.e.8) from 1955. We get (1955 - (8 x 5) = 40 = 1915). So 1915 is the net remainder

```
          6 5 : 2 4 8 2 9 6 : 8 5 :
   2 3              1 12 25 28 28 195
              ___________________________
                    1 0 4 9 8/1 9 1 5
```

Q = 1 0 4 9 8
R = 1 9 1 5

Even these simple steps can all be performed just with mental working along with a very little rough work and the quotient and remainder obtained in a single step. Let us practice the process by doing the mental working in the following illustrations. Further in next few examples, it is more convenient to work with a single flag digit.

32.3.4 ILLUSTRATION:8

Divide 26786 by 114

```
 1 1 4 ÷  2 6 7 8 6
       4 : 2 6 7 8 : 6
  1 1        4 6 12
       ------------------
          2 3 4  : 1 1 0:
```

Therefore Q = 2 3 4

R = 1 1 0

32.3.5 ILLUSTRATION:9

see 32.4

Divide 342945 by 136

```
       6 : 3 4 2 9 4 : 5
  1 3       8 5 3 9
       -----------------
          2 5 2 1 : 8 9:
```

Therefore Q = 2521

R   89

32.3.6 ILLUSTRATION:10

Divide 646275 by 154

```
        4 : 6 4 6 2 7:5
  1 5       4 15 13 11
        ----------------
            4 1 9 6: 9 1:
```

Hence

Q = 4196

R = 91

32.4 Special Cases : In this process of division sometimes we find that the gross remainder left over is too small the necessary adjustment product is bigger than it. This problem is easily handled by just using the next lower digit as our new quotient digit. This trial and adjustment situation is equally encountered even in the division in the present method. The following example illustrates this facet.

32.4.1 ILLUSTRATION 11: Divide 601425 by 76

$$\begin{array}{l} {}_{7}6 : 6\ 0\ 1\ 4\ 2 : 5 : \\ \qquad\quad 11\ 6\ 3\ 5 \\ \hline \qquad\quad 7\ 9\ 1\ 3 : 3\ 7 : \end{array}$$

Here in the first division by 7 we can put down 8 as a quotient digit, but the remainder then left (4) will be too small for the subtraction expected at the next step (48). So we take 7 as the quotient digit and prefix the remainder 11 to the next dividend digit and then proceed as usual.

32.5 Decimal form: The Vedic DHAWAJKAM METHOD can be easily extended to get the result in decimal form by continuing the process of division of the remainder. We place the additional zeros after the dividend. The following examples shall illustrate the process.

32.5.1 ILLUSTRATION 12: Divide 8352 by 79 to three places of Decimals.

$$\begin{array}{l} {}_{7}9 : 8\ 3\ 5 : 2.\ 0\ 0 \\ \qquad\quad 14\ 10\ \ 83 \\ \hline \qquad\quad 1\ 0\ 5 : 7\ 2\ 1 \\ \hline \end{array}$$

The process of solving these questions is same. We put the decimal point as soon we start the division of the remainder.

32.5.2 ILLUSTRATION 13 : Divide 711.023 by 38 to three places of decimals

$$\begin{array}{l} {}_{3}{}^{3}8 : 7\ 1\ 1.\ 0\ 2\ 3 \\ \qquad\quad 49\ 6\ 11 \\ \hline \qquad\qquad 1\ 8.\ 711 \end{array}$$

32.5.3 ILLUSTRATION 14 : Divide 221 by 52 to three places of decimals.

```
   2 : 2 2 1. 0 0
5
         2 3 1
   ______________
         4. 2 5 0
   ______________
```

32.5.4 ILLUSTRATION 15 : Divide 7.5 by 54 to 5 places of decimals.

```
4 : 7. 5 0 0 0 0
    2 6 8 8 8
  ______________
    . 138888
  ______________
```

CHAPTER 33

DIVISION CHECK

वैदिक गणित अध्ययन

33. 1. Introduction

The simple 'Vedic Mathematics' procedure of checking the correctness of the multiplication results can easily be extended to check the calculations of any division of numerical numbers.

Further, it is really interesting to observe that the extremely simple Vedic Method of checking the calculations of any numerical division can be easily extended even to the algebraic division. This process is equally applicable to the problems where remainder is left over.

33. 2. Procedure

It shall be interesting to explain the rather simple, super fast and accurate checking procedure, with the help of an example.

Illustration 1: Divide 46956 by 65.

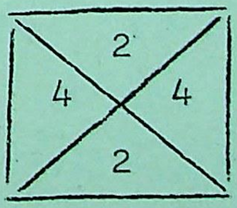
Following the straight division process, the result is obtained in two lines (refer Chapter 32. The quotient is obtained as 722 with a remainder of 26.

Step I: Subtract the remainder from the dividend (46956-26=46930 is exactly divisible by the divider 65 in this case). The result obtained is exactly divisible by the divider.

Step II: By simple observation we can conclude that if our answer of division is correct. The modified dividend obtained in Step I should be equal to the multiplication of the quotient with the divisor.

Therefore

$$\begin{array}{r} 7\ 2\ 2 \\ \times\ \ 6\ 5 \\ \hline 46930 \\ \hline \end{array}$$



2 x 2 = 4

Step III: The problem at this stage has become exactly the same as has been detailed out in Multiplication Check' (Chapter 34):

Therefore the exact checking procedure can be straight way applied.

(a) 7 2 2; 7 + 2 + 2 = 11; 1 + 1 = 2 (Upper triangle No.)

6 5 ; 6 + 5 = 11; 1 + 1 = 2 (Lower triangle No.)

Therefore the right hand side triangle No. is 2 x 2 = 4.

(b) 4 6 9 3 0 ; 4 + 6 = 10; 1 + 0 = 1; 1 + 9 = 10; 1 + 0 = 1;
1 + 3 = 4; 4 + 0 = 4

Therefore the left hand side triangle no. is obtained as 4. The simple straight forward and superfast vedic method of checking the division has been satisfied.

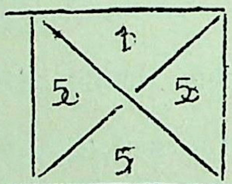
33.2.1 Example 2 : Divide 876456 by 68

By applying the straight division process we get the quotient as 12889 with a remainder of 4.

Step 1: Subtract the remainder from the dividend
(876456 - 4, = 876452)

Step 2: The modified dividend obtained in Step I should be equal to the multiplication of the quotient with the divisor, i.e.

```
 1 2 8 8 9
 x     6 8
 ---------
  876452
```



1 x 5 = 5

Step 3: The problem at this stage has become exactly the same as has been detailed out in multiplication check.

(a) 12889 ; 1 + 2 + 8 + 8 + 9 = 28, 2 + 8 = 10, 1 + 0 = 1;
(upper triangle number) = 1

(b) 68; 6 + 8 = 14, 1 + 4 = 5
(lower triangle number

Therefore the R.H.S. triangle number is 1 x 5 = 5.

(c) 876452; 8 + 7 = 15, 1 + 5 = 6;
6 + 6 = 12, 1 + 2 = 3;
3 + 4 + 5 = 12, 1 + 2 = 3;
3 + 2 = 5.

Therefore the L.H.S. triangle number is 5.

Step 4: 5 = 5 . So our answer is correct.

33.2.2 Example 3: Divide 45696789 by 75.

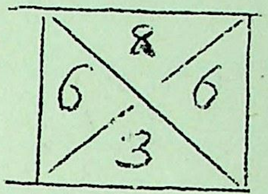
By applying the straight division process we get the quotient as 609290 with a remainder 39.

Step 1: Substract the remainder from the dividend

(45696789 - 39 = 45696750)

Step 2: The modified dividend obtained in Step 1 should be equal to the multiplication of the quotient with the divisor i.e.

$$\begin{array}{r}609290\\ \text{x}\quad 75\\ \hline 45696750\\ \hline\end{array}$$



Step 3: The problem at this stage has become exactly the same as that of 'Multiplication check'.

(a) 609290; 6+2 = 8 (upper triangle number)

(b) 75; 7 + 5 = 12, 1 + 2 = 3 (lower triagle number)

Therefore the right hand side triangle number is

8 x 3 = 24, 2 + 4 = 6.

(c) 45696750; 4 + 5 + 6 = 15, 1 + 5 = 6; 6 + 7 = 13,

1 + 3 = 4; 4 + 5 = 9; 9 + 6 = 15, 1 + 5 = 6.

Therefore theL.H.S. triangle number is obtained as 6.

Step 4: 6 = 6. So our answer is correct.

33.3 Algebraic Division

Even the correctness of the division of algebraic equations can be easily checked by using the same Vedic Sutra.

The 'Vedic Mathematics' procedure outlined above for checking the division of numerical number is exactly applicable for the algebraic division as well. Only a small simple additional step is required for transforming the algebraic problem into a numerical problem.

Step 1: We assign the value of x equal to 1. The assigned value of x (1) is substituted in all the algebraic expressions appearing in the division process and simplified. The four expressions correspond to (i) divisor, (ii) Dividend, (iii) Quotient and (iv) Remainder.

Step 2: Once the numerical values of the above four expressions have been computed, the remaining process is the exact repitition of the very simple and magic speed procedure outlined in previous section.

33.3.1 Example 4: Divide $5x^4 + 7x^3 - 3x^2 + 15x + 24$ by $x^2 - 2x + 3$

Following the steps as illustrated in previous chapter we obtain the quotient as $5x^2 + 17x + 16$ and the remainder equal to $-4x - 24$ in two lines.

Step 1: Substitute $x = 1$ and compute the numerical value.

The dividend $E = 5x^4 + 7x^3 - 3x^2 + 15x + 24$

$E = 5 + 7 - 3 + 15 + 24$

$\therefore D = 48$

The divisor $D = x^2 - 2x + 3$

$= 1 - 2 + 3 = 2.$

The quotient $Q = 5x^2 + 17x + 16$

$= 5 + 17 + 16 = 38.$

and the remainder $R = -4x - 24 = -4 - 24 = -28.$

Step 2: Therefore, the modified dividend is $E' = E - R$

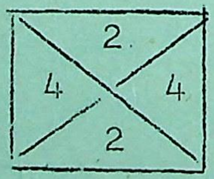
$= 48 - (-28) = 48 + 28$

$= 76$

(refer Step 1 of the previous section 2)

Therefore, the division checking problem of algebraic expressions simply reduces to a simple multiplication checking problem at this stage.

$$\begin{array}{r} 38 \\ \times\ 2 \\ \hline 756 \end{array}$$



$2 \times 2 = 4$

Step 3: Further following the procedure already explained (refer item 2). Top triangle value 38; $3 + 8 = 11$, $1 + 1 = 2$

Bottom triangle value 2.

$\therefore$ $2 \times 2 = 4$ is the R.H.S. triangle value.

L.H.S. triangle $\therefore$ 76; $7 + 6 = 13$, $1 + 3 = 4$.

The number in L.H.S. & R.H.S. triangles are equal. Hence Vedic Check of multiplication is satisfied, as much indirectly we have checked the division of even algebric expressions.

It is enteresting to note that, otherwise there is no direct, short, quick and yet certain, checking method available for the algebric division problems.

33.3.2 Example 5: Divide $9x^4+5x^3-7x^2+18x+32$ by x^2-2x+3.

We obtain the quotient as $9x^2+23x+12$ and the remainder equal to $-27x - 4$

CHECK

Step 1: On putting $x = 1$

The dividend $E = 9x^4 + 5x^3 - 7x^2 + 18x+32$

$= 9 + 5 - 7 + 18 + 32 = 57$

The divisor $D = x^2 - 2x + 3 = 1 - 2 + 3 = 2$

The Quotient $Q = 9x^2 + 23x + 12 = 9 + 23 + 12 = 44$

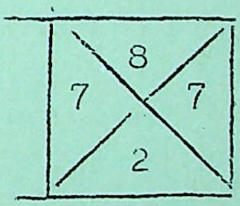
and the remainder $R = 15x27x67\ 4 = -31$

Step 2: The modified dividend as $E' = E - R$

$= 57-(-31) = 57+31 = 88$

Now it is just a multiplication checking problem.

44	65
x 2	x 2
88	130



Step 3: Therefore, top triangle = 44; 4 + 4 = 8

Bottom triangle = 2

∴ 8 x 2 = 16, the R.H.S. triangle value = 1 + 6 = 7

L.H.S. triangle ∴ 88, 8 + 8 = 16, 1 + 6 = 7.

Here 7 = 7. Hence our answer is correct.

33.3.3 Example 6: Divide $3x^4 - 2x^3 - 3x - 2$ by $x^2 + 1$

We obtain the quotient $3x^2 - 2x - 3$ and the remainder is $\bar{1}x + 1$.

Step 1: On putting $x = 1$.

The dividend $E = 3x^4 - 2x^3 - 3x - 2 = 3 - 2 - 3 - 2$

$= -4 = \bar{4}$

The divisor $D = x^2 + 1 = 1 + 1 = 2$

The quotient $Q = 3x^2 - 2x - \underline{3} = 3 - 2 - 3 = \bar{2}$

The remainder $R = \bar{1}x + 1 = \bar{1} + 1 = 0$

Step 2: The modified dividend is $E' = E - R$

$= \bar{4} - 0 = \bar{4}$

Using simple multiplication checking problem

$$\begin{array}{r} -\ 2 \\ \times\ 2 \\ \hline -\ 4 \end{array} \quad = \bar{4}$$

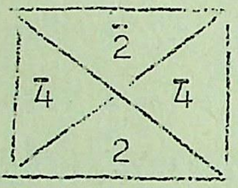
Step 3: Top triangle value $= \bar{2}$

Bottom triangle value $= 2$

$\therefore\ \bar{2} \times 2 = \bar{4}$ the R.H.S. triangle.

L.H.S. triangle $\therefore\ \bar{4}$

Here $\bar{4} = \bar{4}$. Hence our answer is correct.



$\bar{2} \times 2 = \bar{4}$

CHAPTER 38

VEDIC NUMERICAL CODE

38.1 Introduction

It may be a matter of interest to note that Vedic scholars of ancient time did not use figures (for big numbers) in their numerical notations but preferred to use Sanskrit alphabet to represent various numbers. This was perhaps done to facilitate the mathematical knowledge stored and transferred on to subsequent generations by way of language, understood to mankind. Thus presenvation of their knowledge was better managed and understood by subsequent followers. Sutras and Sanskrit verse are in fact treasures of such kind which can be easily decoded and assimilated even to the modern world.

38.2 Recent Research

It is interesting to observe that the latest research of the modern scientists is once again bringing before humanity the perfections of the sanskrit language and lot of systematic effort is being done, the world over, to unearth and decode the scientific knowledge contained in the ancient sanskrit texts with special reference to THE VEDAS. In this context, it is interesting to point out the honour being given to the sanskrit language by the American Association for Artificial Intelligence. The artificial intelligence is the special research area of computers used for the robots - the mechanized servants. The full cover page of the AI magazine (volume 6 No.1 spring 85), an official publication of the American Association for the artificial intelligence is decorated with sanskrit shlokas. This particular issue also contains a research article of 10 pages by Dr. Rick Briggs, who is a computer scientist at NASA, American Space Research Centre, CALIFORNIA, on Knowledge representation in Sanskrit and Artificial Intelligence. Dr. Rick Briggs says, there is a wide spread belief that Natural languages are unsuitable for the transmission of many ideas that artificial languages can render with great precision and mathematical rigor. But this dichatomy which has served as a premises underlying much work in the area of linguistics and artificial intelligence, is a false one. There is at least one language Sanskrit which for the duration of almost 1000 years was a living spoken language with a considerable litrature of its own. Besides works of litrery value, there was a long philosophical and grammatical tradition that has continued to exist with undiminished vigour until the present century. Among the accomplishments of the grammarians can be reckoned a method for paraphrasing Sanskrit in a manner that is identical not only in essence but inform with current work in artificial intelligence. This article demonstrates that a natural language can serve as an artificial language also, and that much work in AI has been reinventing a wheel millenia old. It is interesting to speculate as to why the Indians, found it worthwhile to pursue studies into unambiguous coding of natural language into semantic elements. It is tempting to think of them as computer scientists. Let us not forget that among the great accomplishments of the Indian thinkers were the invention of zero and of the binary number system a thousand years before the west reinvented them. It will be useful to point out here that all the calculators and computers of the world are based on the binary number system. Further the computers are also using the vedic Vinculum system in there IC circuits .

38.3 THE CODE:

The code system in hindi could be written down as:

क	र	प	य --	1
ख	6	फ	र	2
ग	ड	ब	ल	3
घ	ढ	भ	व	4
ड	ण	म	शं	5
च	त	-	ष	6
छ	घ	-	. स.	7
ज	द	-	ह	8
झ	ध	-	-	9
	क्ष और सून्यम		-	0

Note: Further and are also be coded as 5

38.3

- 0

This code system can also be written in English as follows:

(i) ka, tta, pa and ya all denotes 1,
(ii) kha, dah, pha, ra all denotes 2,
(iii) ga, da, ba, la all denotes 3,
(iv) gha, dha, bha, va stands 4,
(v) gna, na, ma and sha denotes 5,
(vi) ca, ta and ksha denotes 6,
(vii) cha, tha, sa denotes 7,
(viii) ja, da and ha stands 8,
(ix) jha, dah denotes 9
(x) Ksa (or Ksudra or shunyam) for 0

The vowels are ignored and in case of conjunct consonants, the last consonant is alone counted.

38.4 Illustration:

Thus pa pa is 11, ma ma is 55, ta ta is 11, ma ry is 52 and so on.

It was left to the author to select proper consonant or vowel. This gave lattitude to the writers to frame the language of their writings to acquire more than one meanings. For instance, kapa, tapa, papa and yapa all mean 11.

38.5 Applications.:

We now take four illustrations which when decoded provide interesting mathematical insight:

(i) केवलैः सप्तकं गुण्यात्

Here Saptakam means 7 and Kevalaih represents 143. We are told that in case of 7 our multiplicand should be 143.

Applying Ekanyuna corollary of Nikhilam all this means as for 7 (as denominator) multiply 143 by 999. This is the answer for -

$$\frac{1}{7} = .\dot{1}4285\dot{7}$$

(ii) कलौ क्षुद्रससैः

Here kalau means 13 and Khudrasasaih represents 077.

The hint here is that for $\frac{1}{13}$ multiplicand is 077, i.e.

$\frac{1}{13}$ is obtained by 077 x 999

Thus $\frac{1}{13} = 0.\dot{0}\,7\,5\,9\,2\,\dot{3}$

(iii) कंसे क्षाभदाहखलैर्मलैः

Here Kamse means 17 and ksama-daha-khalairmalaih means 0 5 8 8 2 3 5 3. Thus

$\frac{1}{17}$ is obtained by 0 5 8 8 2 3 5 3 x 9 9 9 9 9 9 9 9

i.e. $\frac{1}{17}$ = . 0 5 8 8 2 3 5 2 9 4 1 1 7 6 4 7

(iv) Lastly a Shloka for obtaining value of π/10.

गोपी भाग्य मधुव्रात - शृङ्गिशोदधि सान्धिग
खल जीवित खाताय गलदाला रसंधर ।।

This Shloka has two sets of literal meaning, first in praise of Lord Krishna and second for the Lord Shiva. Third meaning (after decoding) gives value of π/10 to 32 places of decimals as below -

$\frac{\pi}{10}$ = .31415926535897932384626433832792 --------

38.6 Observations:

Dr. V.P. Dalal (of Heidelburg University Germany) after knowing above Shloka's merit commented as follows -

It shows how deeply the ancient Indian Mathematicians penetrated, in the subtlety of their calculations, even when the Greeks had no numerals above 1000 and their multiplications were so very complex, which they performed with the help of the counting frame by adding so many times the multiplier; 7 x 5 could be done by adding 7 on the counting frame 5 times. etc. etc.

38.7 Further research by SSG is revealing very interesting results. For instance the secred Gaytri Mantra, when decoded yields the secred 108 number. Further details shall be presented in Part II of the c. course.

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

IA

Tutorial 1
VEDIC BASE SYSTEM

वैदिक गणित अध्ययन

1. Choose the appropriate base for the given nos. and write them in the Vedic Base System style.

(i) 10005 (ii) 999876 (iii) 8
(iv) 100 (v) 97 (vi) 123
(vii) 1345 (viii) 872 (ix) 1000001
(x) 999

2. Choose appropriate, base, sub base wherever required, and write in the Vedic style.

(i) 52 (ii) 49989 (iii) 899
(iv) 256 (v) 3003 (vi) 24937
(vii) 9827 (viii) 2700 (ix) 287
(x) 507

Note: These problems are taken for taking the submultiples of 1/2, 2, 3, 4, 5, 1/4 etc.

3. Write 232 taking (i) 100 as base and 200 as sub-base
(ii) 1000 " " 200 " "
(iii) 1000 " " 250 " "

Note: This problem illustrates that one no. can be represented by using different bases and sub bases.

4. Check the errors

No.	Base	Sub-base	Submultiple	Vedic style
(i) 256	1000	250	2.5	250+006
(ii) 303	100	300	3	303-3
(iii) 999	1000	x	x	1000-1
(iv) 9996	100	x	x	9996-0004
(v) 196	100	200	2	196 -04
(vi) 50016	1000000	50000	1/2	50016-16

In problem (i) of Question 4 it appears that 256 should be represented as 250+6. i.e. [illegible] submultiple of 2.5, which
Also we can think of [illegible] used.
quite inconvenient and hence not used.

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

5. Find out the base and correct number in the following and also write the sub-base wherever necessary.

Vedic style of writing	Deviation	Base	Correct Number
19 - 1	- 1		
98 - 2	- 2		
105 + 5	5		
198 - 2	- 2		
178 - 22	-22		
4955 - 45	-45		
225 + 25	25		

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

वैदिक गणित अध्ययन

TUTORIAL OF -2/1

CHAPTER -2 (TABLES)

1. Write down the tables of following numbers upto 25th level, using Vedic approach:

(i) 8 (ii) 18 (iii) 13

(iv) 32 (v) 102 (iv) 123

(vii) 211 (viii) 92 (hint: 92=1x100)-1x(10) +2x(1)

(ix) 182

(x) 998 (hint: 998 = 1x(1000)+0x(100)-0x(10) -2x(1)

Q. 2. Find out the operator numbers for the following numbers and then write down the tables upto tenth level:

(i) 88 (ii) 279

(iii) 312 (iv) 1121

(v) 1819

Q. 3. Write down the tables of all the illustrations given in the text upto 30th level.

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

TUTORIAL

CHAPTER 3 (VINCULUM)

Q. 1. Write the following numbers in Vinculum form:

(i) 298 (ii) 63 (iii) 971

(iv) 1618 (v) 298706 (vi) 191816

(vii) 48990291 (viii) 90836281

(ix) 99910892 (x) 909125468 2

(Hint: Use of Nikhilam sutra will help you)

Q. 2. Write down the Normalized form of following vinculum numbers.

(i) $2\bar{7}1$ (ii) $7\bar{1}\bar{2}8$ (iii) $3\bar{5}\bar{1}\bar{3}$

(iv) $1\bar{2}\bar{3}1\bar{1}\bar{1}1$ (v) $2\bar{7}2\bar{1}6\bar{5}3$

(vi) $3\bar{8}\bar{9}\bar{6}0\bar{1}\bar{3}4$ (vii) $5\bar{1}\bar{9}\bar{2}3\bar{3}\bar{2}1$

(viii) $5\bar{1}\bar{2}0\bar{1}\bar{2}06$ (ix) $5\bar{4}1\bar{3}2\bar{2}1\bar{1}0$

(ix) $5\bar{1}7\bar{2}3\bar{3}5\bar{4}3$

Q. 3.(a) What is your opinion & impression about this new Vinculum approach ?

(b) Do you know that this is used in computers? How this has straight gone to computers?

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T-10.1

TUTORIAL 10

वैदिक गणित अध्ययन

[illegible]

Q.1 Compute the following multiplications:

(i) 12 x 12 (ii) 14 x 13 (iii) 17 x 17
(iv) 108 x 109 (v) 102 x 103 (vi) 112 x 108
(vii) 113 x 109 (viii) 112 x 111 (ix) 1002 x 1009
(x) 1007 x 1012 (xi) 1110 x 1004 (xii) 1003 x 1002
(xiii) 19 x 17

Q.2 Perform the following multiplication by using Nikhilam Sutra :

(i) 9 x 7 (ii) 8 x 7 (iii) 7 x 6 (iv) 6 x 5
(v) 91 x 93 (vi) 92 x 95 (vii) 87 x 97 (viii) 86 x 98
(ix) 58 x 99 (x) 67 x 96 (xi) 87 x 91 (xii) 598x998
(xiii) 99981 x 99998

Q.3 Obtain the Vinculum-portion' and use the Nikhilam Sutra to perform the following multiplications:

(i) 11 x 9 (ii) 14 x 6 (iii) 15 x 6 (iv) 111 x 98
(v) 102 x 98 (vi) 125 x 95 (vii) 1025 x 997
(viii) 1034 x 998 (ix) 1002 x 92 (x) 1003 x 97
(xi) 100007 x 9998 (xii) 10008 x 9992 (xiii) 1000[illegible]x9993

वेदात् सर्वं प्रसिद्धयति।
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

TUTORIAL 12

MULTIPLICATION

Perform the following multiplications. Indicate the Sutra or subsutra in each case.

Q.1. Squaring Numbers

93 x 93 91 x 91 98 x 98 101 x 101
133 x 133 111 x 111 988 x 988 991 x 991
993 x 993 994 x 994 977 x 977 999 x 999
9997 x 9997 9991 x 9991
17 x 17 19 x 19 21 x 21 103 x 103
106 x 106 109 x 109 111 x 111 113 x 113
1009 x 1009

Q.2 35 x 35 55 x 55 75 x 75 105 x 105
125 x 125 165 x 165

Q.3 47 x 47 62 x 68 81 x 89 74 x 76
91 x 99 103 x 107 112 x 118 111 x 119
104 x 106 191 x 109 194 x 106 692 x 608
889 x 813 893 x 807 897 x 803

Q.4 7 x 9 6 x 9 8 x 9 14 x 99
11 x 9 17 x 99 767 x 999 9367 x 9999
8437 x 9999 9 x 99 63 x 999 767 x 99999

Q.5 Find the square of

(i) 13 (ii) 1003 (iii) 96

(iv) 9889 (v) 14 (in this the R.H.S. comes out to be of two digits)

Note: In (ii) the 3^2 comes out to be 9 i.e. single digit. We've to prefix 0's to get the correct answers.

(vi) 134 (In this we have to transfer two digits from R.H.S.

(vii) 52 (This is far from base so we can use a sub sutra

(viii) 496 (ix) 256 (x) 249 (xi) 202

(xii) 1969

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T-13.1

वैदिक गणित अध्ययन

TUTORIAL 13

GENERAL FORMULA FOR MULTIPLICATION

1. Solve the problems of T10 and T12 by the general method, alongwith other applicable methods. Compare and find out the most convenient sutra in each case.

Solve Q.2 and 3 using URDHAVA method.

2. i) 21 x 63 (ii) 27 x 67 (iii) 33 x 73
iv) 133 x 179 (v) 141 x 281 (vi) 371 x 433
vii) 1278x 9034 (viii) 12078x 10304 (ix) 378^2
x) 6471x 6012 (xi) 81265x 32112

3. i) 89588 x 787 (ii) 396 x 914
iii) 8858 x 78 (iv) 6548 x 787
v) 432008x 1212 (vi) 58010 x 7017
vii) 9238 x 1010101
viii) 880001 x 061010

4. Solve following using viniculum approach by general Sutra:-
i) 787 x 391 (ii) 282 x 132
iii) 181 x 98 (iv) 271 x 87
v) 591 x 187 (vi) 483 x 179
vii) 1892 x 197 (viii) 9867 x 918
ix) 90908x81826 (x) 81739 x 9019

5. What is the difficulty you are facing in these problems. Are you finding any other method for its simplification. If yes then solve your problem by that method.

6. Solve the problems by the help of Nikhilam and Urdhava both and compare
799 x 398, 5439 x 4569, 177 x 165
6789 x 9876

TUTORIAL 14

MULTIPLICATION CHECK

1. Check your answers of Q.2 of Tutorial 13 by this Vedic Approach of Multiplication check.

2. Are the following multiplication correct. Write True/False. Use multiplication check:

i) 96 x 98 = 9408 (ii) 879 x 998 = 886082

iii) 1016 x 1006 = 1122096

iv) 1033 x 977 = 1029901

v) 87965 x 99988 = 8796324070

vi) 128 x 672 = 86016 (vii) 635 x 502 = 318740

viii) $989^2 = 987121$ (ix) $775^2 = 601625$

x) 1021 x 2103= 2147163 (xi) 582 x 231 = 134542

xii) 87265 x 32117 = 2802690005

xiii) 817 x 332 = 271244

xiv) 1234567809
x 9999999999
1234567808/8765432191

xv)))))))))

xv) 999999997
x 999999981
999999878/000000057

आध्यात्मिक अध्ययन संस्था (आअस)

SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

TUTORIAL 15

RECURRING DECIMALS

1. Find the values of following in Recurring decimals :

   (i) $\frac{1}{39}$ (ii) $\frac{1}{49}$ (iii) $\frac{1}{89}$

2. Find the inverse of following upto maximum accuracy :

   (i) 19 (ii) 69 (iii) 23

   (iv) 17 (v) 119

3. (i) Find the value of 1/39 upto infinite number of places of decimal.

   (ii) Find the value of 1/29 upto 84 places of decimal.

   HINT: By using recurring decimals by Vedic Approach.

4. Compute 1/13, 1/7 and 1/19 .

5. Comment on the efficiency of the Vedic Method.

आध्यात्मिक अध्ययन संस्था (आडस)

SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T-29

TUTORIAL-29

DIVISION (BY NIKHILAM METHOD)

Q.1 Perform division by Nikhilam Method in the following cases

(i) 12483 ÷ 7988 (ii) 10015 ÷ 89
(iii) 12345 ÷ 8899 (iv) 124567 ÷ 882
(v) 210011 ÷ 8996 (vi) 1010108 ÷ 8988
(vii) 1020017 ÷ 98987

Q.2 (i) Divide 11000 by 88
(ii) Divide 1987 by 899
(iii) Divide 1294468 by 89997
(iv) Divide 11299251 by 99979
(v) Divide 2645678 by 7777
(vi) Divide 789645 by 8787
(vii) Divide 245654 by 9788

Q.3 (i) Divide 145672 by 99 upto 3 places of decimals.
(ii) Divide 1758011 by 887 upto 4 places of decimals.
(iii) Divide 101010 by 23 upto 2 places of decimals.
(iv) Divide 1213212 by 799 upto 3 places of decimals.
(v) Divide 11223311 by 8989 upto 2 places of decimals.
(vi) Divide 10010245 by 78779 upto 2 places of decimals.
(vii) Divide 250000 by 79 upto 3 places of decimals.

वेदात् सर्वं प्रसिद्धयति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है ।
Eternal Source of All Knowledge

TUTORIAL-33

DIVISION CHECK

1. Check your answers of the followings. Use the Vedic checking method for division.

   (i) Problem I of tutorial (29)

   (ii) Problem I, II and III of Tutorial (30)

   (iii) Problem I and III of Tutorial (32)

   (iv) Problem I and II of Tutorial 31 and 34.

2. Write True/False for the following divisions. Use Vedic Division check.

   (i) $\underline{9}/12000212 = 133356/8$ Q/R

   (ii) $888\underline{8}/101020 = 12/3252$

   (iii) $8898\underline{7}/1030007 = 10/40237$

   (iv) $2\underline{3}/1011 = 43/22$

   (v) $798\underline{9}/\ 10102 = 1/2913$

   (vi) $x^2-x-\underline{1}/\ x^4-x^3+x^2+3x+5$

   $\therefore\ Q = x^2 + 12\ ,\ R = 6x + 7$

   (vii) $2x-\underline{4}/\ -4x^3-7x^2+9x-12$

   $\therefore\ Q = -2x^2 - 7\frac{1}{2}x - 13\frac{1}{2}$

   $R = -54$

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आऽस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T-31

वैदिक गणित अध्ययन

TUTORIAL-31

ALGEBRAIC DIVISION

Q.1. Perform following division using paravartaya formula.

(i) $4x^3 - 7x^2 - 7x + 12$ by $2x-4$

(ii) $3x^2 - x - 5$ by $3x - 7$

(iii) $2x^5 - 9x^4 + 5x^3 + 16x^2 - 16x + 36$ by $2x^2-3x+1$

(iv) $x^4 + 4x^3 + 6x^2 + 4x + 1$ by x^2+2x+1

(v) $x^4 - 4x^2 + 12x - 9$ by $x^2 + 2x + 1$

(vi) $16x^4+36x^2 + 16x + 9$ by $4x^3 - 2x^2+2x-3$

(vii) $x^5+ x^3 - 7x^2 + 9$ by $x^3 - 2x^2 + 2x -3$

(viii) $21x^6+7x^5+15x^4+29x^3+x^2+15x+3$ by $3x^3+x^2+3$

Q.2 Write down the answer (i.e. the quotient and the remainder) in following cases by performing mental calculations using paravartaya formula.

(i) $x^3 + 7x^2+9x +11 \div x - 2$; Q = R =

(ii) $x^3 - x^2 -7x +3 \div x - 3$; Q = R =

(iii) $x^4 -3x^3+7x^2+5x+7 \div x - 4$; Q = R =

(iv) $-4x^3-7x^2+9x +12 \div x+1$; Q = R =

(v) $x^4-3x^3 +7x^2+5x+7 \div x+4$; Q = R =

(vi) $6x^4+13x^3+39x^2+37x+ 45 \div x^2-2x-9$; Q = R =

(vii) $2x^4-3x^3+3x - 2 \div x^2+ 1$; Q = R =

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

TUTORIAL-32

STRAIGHT DIVISION

1. Perform division in following cases

   (i) 48986 by 83 (ii) 60998 by 65

   (iii) 1638467 by 138 (iv) 8956425 by 78

   (v) 888888 by 46 (vi) 753142 by 38

   (vii) [illegible]8 by 123.

2. (i) Divide 8475 by 65 upto 3 places of decimals.

   (ii) Divide 750.2532 by 781 upto 4 places of decimals.

   (iii) Divide 80.23 by 243 upto 5 places of decimals.

   (iv) Divide 764256 by 1134 upto 2 places of decimals

   (v) Divide 84246[illegible] by 846 upto 3 places of decimals.

3. Divide following [illegible] straight division method using Nikhilam method [illegible] division and dividend.

   (i) 9168456 ÷ 8567 (ii) [illegible]4563919 ÷ 78

   (iii) 784653 ÷ 649 (iv) [illegible]4652 ÷ 69

4. Divide following by straight division method using table for suitable divisor digit

   (i) 642456 ÷ 39 (ii) 7456 ÷ 41

   (iii) 524567 ÷ 55 (iv) 845 ÷ 89

   (v) 7926778 ÷ 119

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T-30

वैदिक गणित अध्ययन

TUTORIAL-30

PARAVARTAYA METHOD

Q.1 Perform division by paravartaya method in the following cases.

(i) 1235 ÷ 113
(ii) 1248 ÷ 161
(iii) 13291 ÷ 1125
(iv) 12351 ÷ 1012
(v) 42678 ÷ 9818
(vi) 1231 ÷ 838
(vii) 42678 ÷ 829

Hint: Use Nikhilam Sutra for (v, vi and vii)

Q.2 (i) Divide 21356 by 8818 upto 2 decimal places.
(ii) Divide 1245 by 182 upto 3 places of decimals.
(iii) Divide 2534 by 224 upto 4 places of decimals.
(iv) Divide 7586 by 672 upto 3 places of decimals.
(v) Divide 568519523 by 29356 upto 2 decimal places
(vi) Divide 453210 by 2466 upto 3 places of deoimals.
(vii) Divide 65210400 by 333 upto 2 places of decimals.

Q.3 Perform division using Nikhilam Sutra.

(i) 5243256 ÷ 7998
(ii) 345678 ÷ 6007
(iii) 14587819 ÷ 11465
(iv) 2468679 ÷ 24688
(v) 13887900 ÷ 1124
(vi) 2465667 ÷ 9999
(vii) 3456758 ÷ 888

TUTORIAL - 38

1. Decode the following words. Use Numerical code :-

(i) रमेश
(ii) जयदेव
(iii) हरि
(iv) माया
(v) योगेन्द्र
(vi) गाँधी
(vii) सुनील
(viii) दिलीप
(ix) अर्पित
(x) रवि
(xi) कपिल
(xii) मनोहर
(xiii) आदित्य
(xiv) राम
(xv) भारत

2. Find the meaning of the following :-

i) 16 and 56 of 82 are going to 81.
ii) 32 of 4345 taught us English today.
iii) I want my 55 and 22.

3. Find then numbers coded by following words :-

(i) गोपी
(ii) भाग्यम्
(iii) मधुव्रात
(iv) शृङ्गिश
(v) उदधि
(vi) सन्धिग
(vii) खल
(viii) जीवित
(ix) खाताव
(x) गलहाला
(xi) रसंधर

5. Using the words of Q.3 find the value of $\pi/10$ up to 32 places of decimals, decoding the full Shloka.

5. Compose suitable lines/sentences in Sanskrit/Hindi which have simple meaning and also codify the following numerical values according to the Vedic Code.

(i) 39 = 025641
(ii) 29 - 0344827586 2069
(iii) 7 - 143
(iv) 19 - 052631579
(v) 13 - 077
(vi) 17 - 05882353

आध्यात्मिक अध्ययन संस्था (आऽस)　　SPIRITUAL STUDY GROUP (SSG)

वैदिक गणित कार्यशाला

७--१७ अगस्त १९८७

Advanced Vedic Maths

S.No.	Chapter No.	Title
1	1	SQUARE ROOTING
2	2	ALGEBRA-FIRST PRINCIPLES, FACTORISATION
3	3	QUADRATIC AND CUBIC EQUATION
4	4	TRIGNOMETRICAL, HYPERBOLIC FUNCTION AND THEIR INVERSE
5	5	POLYNOMIAL EQUATION
6	6	TRANSCEDENTAL EQUATION
7	7	CALCULUS
8	8	EVALUATION OF DETERMINANTS
9	9	INVERSION OF MATRICES
10	10	CHANGE OF ROOTS OF EQUATION
11	11	THE SOLUTION OF LINEAR AND NON-LINEAR DIFFERENTIAL EQUATION

वेदात् सर्वं प्रसिद्ध्यति

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।

Vedas are the Eternal Source of All Knowledge

CHAPTER 1

SQUARES AND SQUARE ROOTS

Reference: Vertically and Crosswise

1.1 Squaring We make extensive use of the 'duplex', D, for squares and square roots. The term has the following meaning :

For a 1-digit number D is its square,

for a 2-digit number D is twice their product,

for a -3-digit number D is twice the product of the outer pair + the square of the middle digit,

for a 4-digit number D is twice the product of the outer pair + twice the product of the inner pair

and so on.

Thus D(3) = 9, D(42) = 16, D(127) = 18, D(134) = 22.

1.1.1 Example 1 : 43^2 We find the duplexes of , 43 and 3:

$\therefore 43^2 = 16/{}_24/9 = 1849$

1.1.2 Example 2 : 67^2 $36/{}_84/{}_49 = 4489$

1.1.3 Example 3 : 341^2 We find the duplexes of 3,34, , 41 4:

$\therefore 341^2 = 9/{}_24/{}_22/8/1 = 116281$

1.1.4 Example 4 : 1435^2 We find duplexes of 1,14,143,1 , 435,35 :

$\therefore 1435^2 = 1/8/{}_22/{}_34/{}_49/{}_30/{}_25 =$ 20592

1.1.5 Example 5: 21034^2 We find duplexes of 2,21,210,21 ,2103 1034,034, 34, 4:

$\therefore 21034^2 = 4/4/1/{}_12/{}_22/8/9/{}_24/{}_16 =$ 4424 15

If we do the calculation from right to left we can take up the carry figures as we go.

1.1.6 Example 6: 542^2 O , in [illegible] can square from left to right taking up carry figures (to which we add a zero) as we go:

$542^2 = 2{}_59{}_0{}^36{}_67{}_66\ 4$

1.1.7 Example 7: $(x + 3)^2 = x^2 + 6x + 9$ We take duplexes of x, x, x+3, 1.1.8 Example 8 : $(2x - 3y)^2 = 4x^2 - 12\,xy + 9y^2$

1.1.9 Example 9 : $(x+2y+3)^2 = x^2 + 4xy + 6x + 4y^2 + 12y + 9$

1.2 <u>Square Roots</u> This is just the reverse of the squaring process.

1.2.1 Example 10 :

$\sqrt{3\ 8_2 8_4 3_3 7_1 8_1 2\ 4}$

= a b c d

= 6 2 3 2

Here we observe 4 pairs of digits and therefore expect a 4-figure answer if the number is a perfect square. Since th first pair is 38 the first digit is 6, ie. a = 6, and the remainder is $38 - 6^2 = 2$, placed as shown. The next dividend is then 28.[+] The next dividend is 43, and the next duplex is $(2ac+b^2) = 12c+4$. We therefore subtract the 4: 43 - 4 = 39, 39 ÷ 12 = 3 R3, so that c = 3. The next dividend is 37 and the next duplex is (2ad + 2bc) = 12d+ 12, so 37 - 12 = 25, 25 ÷ 12 = 2 R1, therefore d = 2. The next duplex is $(2bd + c^2) = 17$, 18 - 17 = 1; the next is 2cd = 12, 12 - 12 = 0; and the last is $2^2 = 4$, 4-4 = 0, so that those last three duplexes account exactly for the remaining numbers.

1.2.2 Example 11 :

$\sqrt{2\ 8_3 5_5 2_3 6_2 2\ 8\ 1}$

= 5341

This is similar to the example above, and we may notice in both examples that at each step after the first we divide by <u>twice</u> the first digit. This allows the process to be simplified by setting up a chart like that below and dividing at each step by twice the first digit, after subtracting the duplex obtained from the figures which appear <u>after</u> the first figure.

```
        2 8 5 2 6 2 8 1
           3 5 3 2 0 0
10 )       3 5 3
      -----------------
       5/3  4 1.0 0 0
      -----------------
```

i.e. put down 5 R3, then 35 ÷ 10 = 3 R5; put them down. D(3)= 9, 52-9 = 43, 43 ÷10 = 4 R3; put them down. D(34)= 24; 36-24= 12, 12÷ 10= 1 R2; put them down.

[+] The duplex 2ab = 12 b must account for most of this 28 so that b = 2. The remainder, 4, is prefixed to the next figure as shown.

It will be seen that the steps are exactly as in the first method. If we did not know the number was a perfect square and continued the process we would have: D(341) = 22, 22-22 = 0, 0 ÷ 10 = 0 put them down. D(3410) = 8, 8 - 8 = 0, 0 ÷ 10 = 0 put them down. D(34100) = 1, 1-1 = 0, 0 ÷ 10 = 0; and all successive duplexes are zero.

1.2.3 Example 12: $\sqrt{1\ 7\ 9\ 3}$

$$\begin{array}{l} \quad 1\ \ 7\ {}_1 9\ {}_3 3 : {}_5 0\ {}_6 0\ {}_3 0\ {}_{\bar{2}} 0 \\ 8)\ \ \ 1 \qquad 6 \\ \hline \quad 4/2.3\ 4\ 4\ \bar{1}\ldots \\ \hline \end{array}$$

This is not an exact square and so the answer does not terminate.

1.2.4 Example 13 Using the vinculum : $\sqrt{6\ 2}$

$$\begin{array}{l} \quad 6\ 2\ {}_{\bar{2}}0\ {}_{\bar{4}}0\ {}_{7}0\ {}_{0}0\ {}_{\bar{1}}0\ {}_{\bar{2}}0 \\ 16) \\ \hline \quad 8.\bar{1}\ \bar{3}\ 4\ 0\ 1 \\ \hline \end{array}$$

Here, as 62 is close to 64(a perfect square) we may take the first figure as 8 and have a remainder of $\bar{2}$.

1.3 Square Roots of Polynomials

1.3.1 Example 14 : Obtain $\sqrt{4 + 12x + 13x^2 + 6x^3 + x^4}$

As a power series in x.

Solution

$$\text{Answer:} \quad \begin{array}{c|l} 4 & 4 + 12x + 13x^2 + 6x^3 + x^4 \\ \hline \pm & (2/ + 3x + 1x^2 + 0x^3 + 0x^4 \end{array}$$

Check : $(2 + 3x + x^2)^2 = 4 + 12x + 13x^2 + 6x^3 + x^4$

The procedure is the same as for determining the square root of a number, except that there is no carrying. The steps are

1. $\sqrt{4} = 2$

This leads to one solution, and $-\sqrt{4} = -2$ leads to the second. The 2 is written down as the first term in the answer.

2. Doubling this 2, we obtain the divisor, 4. A diagonal stroke now separates this 2 from those ensuing terms which contribute to the duplex (viz 3^2, 2.3.1,
2.3.0 + 1^2, 3.0 + 1.0 etc.)
3. The second term in the answer is given by 12 ÷ 4 = 3.
4. 13 − 3^2 = 4
and 4 ÷ 4 = 1, which is the third term in the answer.
5. The duplex is now 2.3.1. = 6,
and 6 − 6 = 0, so that
0 ÷ 4 = 0 (fourth term of the solution)
6. The duplex is 2.3.0 + 1^2 = 1,
and 1 − 1 = 0, whence
0 ÷ 4 = 0 (fifth term of the solution)
All subsequent terms are found to be zero, and so the solution is exact.

1.3.2 Example 15 : Obtain $\sqrt{4 + 12x + 13x^2 + 7x^3 + 2x^4}$ as a power series in x.

The first three terms being the same as in the previous example, the working starts off the same. The working and solution proceeds thus :-

$$\begin{array}{ll} 4 & \underline{4} + \underline{12x + 13x^2 + 8x^3 + 4x^4} \\ \hline \text{Solution:} \quad & 2 / + 3x + x^2 + \frac{1}{2}x^3 + 0x^4 - \frac{1}{4}x^5 + \frac{5}{16}x^6 + \ldots\ldots\ldots \end{array}$$

The procedure for obtaining the square root in descending powers of x is the same, except that the coefficients are considered in the reverse order.

1.3.3 Example 16 : Obtain $\sqrt{(4x^4 + 8x^3 + 13x^2 + 12x + 4)}$ as a descending power series in x.

$$\begin{array}{ll} 4 & \underline{4x^4} + \underline{8x^3 + 13x^2 + 12x + 4} \\ \hline & 2x^2 / + 2x + \frac{9}{4} + \frac{3}{4x} - \frac{65}{64x^2} + \frac{11}{64x^3} + \ldots\ldots\ldots\ldots \end{array}$$

In general, the square root of a polynomial leads to an infinite series.

EXAMPLES

Q. 1. Square the Following

(i) 43 (ii) 73 (iii) 234 (iv) 712 (v) 382

(vi) 4172 (vii) 6047 (viii) 63213 (ix) 34356

(x) 491928

Q. 2. Find the square root

(xi) 283024 (xii) 4489 (xiii) 670761 (xiv) 60516

(xv) 528529 (xvi) 114244 (xvii) 10784656

(xviii) 55457809. (xix) 7890481 (xx) 3936256

Q. 3. Find the square root to 3 decimal place:

(xxi) 2828 (xxii) 3333 (xxiii) 2122 (xxiv) 234

(xxv) 80908 (xxvi) 8 (xxvii) 10 (xxviii) 6.6

(xxix) 48.81 (xxx) 1.884

Q. 4. Evaluate the following

(i) $\sqrt{x^4 + 4x^3 + 10x^2 + 12 + 9}$

(ii) $\sqrt{x^4 + 6x^3 + 8x^2 + 9x + 3}$

(iii) $\sqrt{x^6 + 2x^5 + 6x^4 + 5x^3 + 2x^2 + x - 2}$

(iv) $\sqrt{x^6 + 3x^5 + 4x^4 + 2x^3 + x^2 + 4x^4 + 2x^3 + x^2 + 5x + 1}$

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

वैदिक गणित अध्ययन

ALGEBRA: FIRST PRINCIPLES; SIMPLE EQUATION AND FACTORIZATION

This chapter is taken from the contents of chapter 6 of Introductory lectures on vedic Mathematics by A.P.Nicholas, J. Pickles and K. Williams published by the Vedic Maths Research Group 106 Mercers Road, LONDON, N-19, with due permission of the Research Group.

2.1 INTRODUCTION

Starting with his nine given elements, the numbers 1-9, the vedic mathematician rapidly finds a need to extend the system. If we add 2 and 3, the answer is 5, which is one of the nine elements. But we have only to add 9 and 1, and it appears that nine elements no longer meet the need.

The situation is resolved by the introduction of a new unit. We might measure short distances in feet, but greater ones in miles; a change of units is found to be convenient, and in consequence the same nine number-elements that were encountered initially find a renewed and more powerful application.

Changing units might involve a factor of 12, as with feet to inches; it might involve a factor of 100, as with pounds to pence; it might involve a factor of 3, as with yards to feet. The rules governing all these cases can be examined simultaneously using the following simple device.

Let the change in units involve an unspecified factor x. e.g. $2x + 3$ could be used to represent 2 feet 3 inches (in which case $x = 12$), or 2 lbs 3 ozs (in which case $x = 16$). This simple step gives rise to the study of algebra. And because algebra springs out of a generalisation of arithmetic, the same laws govern both subjects. By the introduction of an x or a y, we are acknowledging our freedom to choose what we will regard as unity. This quality, of being constantly in a position to start a fresh, from the beginning, gives algebra an enormous potential. Indeed much of modern mathematics springs from algebra.

A difference between algebra and arithmetic is in the manner of handling factors such as 2x + 3 and 4x + 7. In arithmetic it is at this point that positional notation comes into play.

E.g. $47 = 4 \times 10 + 7$ (decimal notation)

or $47 = 4 \times 12 + 7$ (duodecimal notation)

Also, $5\bar{3} = 5 \times 12 - 3$ (duodecimal notation

And $6x - 12 = 6 \times 10 - 12$, on putting $x = 10$

$= 6_{1}\bar{2}$

$= 4\ 8$

The latter pair of examples draw attention to the freedom of viewpoint encountered in Tirthaji's system. We are not obliged to keep units positive, nor to keep them below the base: to have 12 hundreds, 23 tens, and 15 units is acceptable - although we might finally call this collection 1 thousand, 4 hundreds, 4 tens, and 5 units.

If we used a base of x, the concise positional notation met in arithmetic would become available for factors and quadratics etc., and we could write:-

$$2x + 3 = \begin{array}{c|c} x & 1 \\ \hline 2 & 3 \end{array}$$

$$5x - 7 = \begin{array}{c|c} x & 1 \\ \hline 5 & 7 \end{array}$$

$$4x^2 + 6x + 2 = \begin{array}{c|c|c} x^2 & x & 1 \\ \hline 4 & 6 & 2 \end{array}$$

Use is effectively made of this possibility in the parāvartya method of division.

The ensuing lecture is largely concerned with the resolution of quadratic expressions into factors. It is therefore relevant to observe that a quadratic arises from the product of factors, as the following pair of examples show:-

1)
$$\begin{array}{rrr} & 1 & 2 \\ & 1 & 3 \\ \hline 1 & 5 & 6 \end{array}$$

2)
$$\begin{array}{r} x + 2 \\ x + 3 \\ \hline x^2 + 5x + 6 \end{array}$$

First we will consider some first principles applicable to the solution of simple equations.

Then follow a few examples of methods applicable in special cases, for solving quadratic equations.

Finally we will look at direct methods of factorisation of quadratics.

42.2 SIMPLE EQUATION

FIRST PRINCIPLES

There are four simple types of equation which can be solved at sight and which come under the sutra परावर्त्य योज्येत (transpose and adjust). Theoretically all four types are known to the western world, but they are not consistently and universally put into practice as one-line solutions.

42.2.1 TYPE I

1) $5x + 3 = 2x + 7$

$\therefore \quad \underline{\underline{x = 4/3}}$

The intermediate step (carried out mentally) is,

$5x - 2x = 7 - 3$

2) $6x + 7 = 2x + 9$

$\therefore \quad \underline{\underline{x = 2/4 = 1/2}}$

42.2.2 TYPE II

1) $(x + a)(x + b) = (x + c)(x + d)$

$$\therefore\ x(a+b-c-d) = cd-ab$$

$$x = \frac{cd-ab}{a+b-c-d}$$

Here the x^2 terms on both sides cancel out, and on multiplying out we would obtain an equation of Type I. But it is quicker to gather all the constant terms on the right-hand side, and divide by the coefficient of x, as in the second and third lines above.

2) $(x + 2)(x + 5) = (x + 3)(x + 7)$

$$\therefore\ x = \frac{3.7-5.2}{2+5-3-7} = \frac{-11}{3}$$

3) $(x + 3)(x + 2) = (x + 4)(x + 6)$

$$\therefore\ x = -18/5$$

2.2.3 TYPE III

1) $\dfrac{ax + b}{cx + d} = \dfrac{p}{q}$

$$\therefore\ aqx + bq = pcx + dp$$

$$\therefore\ x = \frac{dp - bq}{aq - pc}$$

First we gather constant terms on the right hand side; note the transposition of signs which this entails(÷ becoming x, and + becoming -). Then similarly we gather the terms constituting the coefficient of x on the left-hand side, and these become the divisor.

2) $\dfrac{2x + 5}{x + 3} = \dfrac{4}{5}$

$$\therefore\ x = -13/6$$

3) $\dfrac{3x + 1}{2x + 5} = \dfrac{2}{3}$

$$\therefore\ x = 7/5$$

4.2.2.4 TYPE Iv

1) $$\frac{m}{x + a} + \frac{n}{x + b} = 0$$

$$\therefore x = \frac{-mb-na}{m+n}$$

2) $$\frac{3}{x + 2} + \frac{5}{x + 1} = 0$$

$$\therefore x = -13/8$$

3) $$\frac{2}{x + 3} + \frac{3}{x + 7} = 0$$

$$\therefore x = -23/5$$

4.2.3 SPECIAL CASES

The following are a sample of special types of simple equations mostly quadratics.

First Special Case

4.2.3.1 First Special Case

1) $$x + 1/x = 50/7 = 7 + \frac{1}{7}$$

$$\therefore \underline{x = 7 \text{ or } 1/7}$$

2) $$x + 1/x = 10/3$$

$$\therefore x = 3 \text{ or } \frac{1}{3}$$

3) $$(x+2) + \frac{1}{x + 2} = 26/5$$

$$\therefore x + 2 = 5 \text{ or } 1/5$$

$$\underline{x = 3 \text{ or } -1\ 4/5}$$

4) $$\frac{x + 5}{x + 1} + \frac{x + 1}{x + 5} = 65/8$$

$$\therefore \frac{x + 5}{x + 1} = 8 \text{ or } \frac{1}{8}$$

$$\therefore x = -3/7 \text{ or } -39/7$$

5) $\frac{x+2}{x-1} + \frac{x-1}{x+2} = 5/2$

$\therefore \frac{x+2}{x-1} = 2 \text{ or } 1/2$

$\underline{x = +4 \text{ or } -5}$

6) $x - \frac{1}{x} = \frac{15}{4}$

Here, $\frac{15}{4} = \frac{16-1}{4} = 4 - \frac{1}{4}$

$\therefore x = 4 \text{ or } -\frac{1}{4}$

Note the minus sign in the second solution

7) $x - \frac{1}{x} = \frac{80}{9} = \frac{81-1}{9}$

$\therefore \underline{x = 9 \text{ or } -\frac{1}{9}}$

8) $x + \frac{1}{x} = \frac{13}{6}$

$= \frac{9+4}{6} = \frac{3}{2} + \frac{2}{3}$

$\therefore x = \frac{1}{2} \text{ or } \frac{2}{3}$

9) $x + \frac{1}{x} = 41/20$

$= \frac{16+25}{20} = \frac{4}{5} + \frac{5}{4}$

$\therefore x = \underline{4/5 \text{ or } 5/4}$

A principle underlying these last two examples is brought out in the next example

10) $x + \frac{1}{x} = \frac{a^2 + b^2}{ab} = \frac{a}{b} + \frac{b}{a}$

$\therefore = \underline{\frac{a}{b} \text{ or } \frac{b}{a}}$

2.3.2 Second Special Case

1) $\frac{5x+3}{2x+7} = \frac{4x+9}{7x+5}$

Since $(5x+3) + (4x+9) = (2x+7) = (7x+5)$
$= 9x + 12$,

we can at once say

$\therefore \quad \underline{9x + 12 = 0}$

whence $x = -12/9 = -4/3$

But we are dealing with a quadratic: its second solution can be obtained as follows. Write the equation as

$$\frac{N_1}{D_1} = \frac{N_2}{D_2}$$

Now $N_1 \sim D_1 = (5x + 3) - (2x + 7) = 3x - 4$

and $N_2 \sim D_2 = (7x + 5) - (4x + 9) = 3x - 4$

(Where ~ means that we subtract the smaller term from the larger and for this purpose we can give the coefficient of x a greater weight than the constant term.)

$\therefore$ the second solution is

$\underline{3x - 4 = 0}$

2) $$\frac{5x - 1}{2x + 7} = \frac{8x + 9}{11x + 1}$$

$\therefore \quad 13x + 8 = 0$

i.e. $\underline{x = -8/13}$

and $3x - 8 = 0$

$\underline{x = 8/3}$

3) $$\frac{4x + 5}{2x + 3} = \frac{7x + 9}{9x + 11}$$

$\therefore \quad 11x + 14 = 0$

and $2x + 2 = 0$

$\therefore \quad \underline{x = -14/11 \text{ and } -1}$

4) $$\frac{3x + 7}{3x + 8} = \frac{3x + 12}{3x + 11}$$

$\therefore \quad 6x + 19 = 0$

$\underline{x = -19/6}$

There is no second solution in this instance. If we attempt to find one, we have

$N_1 - D_1 = -1$

$D_2 - N_2 = -1$

and this does not yield a solution. This particular example is really a linear equation in disguise, not a quadratic.

The following is a variation on the type we have been studying:

5) $\dfrac{2x + 3}{11x + 7} = \dfrac{3x + 1}{4x + 5}$

Here $N_1 + N_2 = (2x + 3) + (3x + 1) = 5x + 4$

$D_1 + D_2 = (11x + 7) + (4x + 5) = 15x + 12$

$= 3(5x + 4)$

But we can still say,

$\therefore \; 5x + 4 = 0$

The second solution will be discussed shortly - a different method is now needed.

6) $\dfrac{2x - 1}{5x + 7} = \dfrac{x + 5}{4x + 5}$

Since $(2x-1)+(x+5) = 3x + 4$

and $(5x + 7)+(4x + 5) = 3(3x + 4)$.

$\therefore \; 3x + 4 = 0$

7) $\dfrac{x - 3}{7x + 6} = \dfrac{2x + 4}{5x - 2}$

$\therefore \; \underline{3x + 1 = 0}$

The second solution in these last three examples can be written straight down using as a sub-sutra.

42.3.3 [illegible]

आद्यमाद्येनान्त्यमन्त्येण

i.e. adyamadyenantyamantyena which means, The first by the first and the last by the last.

In this last example, togather the coefficient of x^2 as a single term on the left-hand side of the equation the steps are,

$$\frac{1x - 3}{7x + 6} = \frac{2x + 4}{5x - 2}$$

and we are interested in the coefficients of x^2

$$\begin{array}{ccc} 1 & & 2 \\ - & \times & - \\ 7 & & 5 \end{array}$$

$\therefore$ the coefficient of x^2 is $1.5 - 7.2 = - 9$

Similarly the coefficient of unity is $(-3)(-2) - 6.4 = - 18$

We now divide -9 and -18 respectively by 3 and 1, which are the coefficients of the first factor, $3x + 1$.

Then $\frac{-9}{3} = - 3$ and $\frac{-18}{1} = - 18$, and these are the coefficients of the second factor: i.e. $(-3x - 18)$, or $-3(x+6)$.

Thus $(3x + 1)$ and $(x + 6)$ [or $- 3(x + 6)$] are factors.

Similarly the second solution (for Example 6) is $(x - 10)$, and (for Example 5), it is $(5x + 2)$.

Two more example follow:-

8) $$\frac{2x + 7}{6x + 12} = \frac{5x + 9}{8x + 20}$$

$\therefore$ $7x + 16 = 0$

and $(-2x + 2) = 0$

9) $$\frac{3x - 5}{13x - 1} = \frac{7x + 3}{17x - 5}$$

$\therefore$ $10x - 2 = 0$

and $(-4x - 14) = 0$

The next section illustrates the immense simplicity and power of this latest sutra in a very direct way.

2.4 Factorisation of quadratics

Two sutras are needed

आनुरूप्येण (anurupyena) i.e. proportionately, and

आद्यमाद्येनान्त्यमन्त्येन (adyamadyenantyamantyena), i.e. the first by the first and the last by the last.

E.g. to factorise $2x^2 + 11x + 12$, we note that there are three coefficients, 2, 11, 12. Split the middle of these into two parts in such a way that the ratio of the first coefficient to the first part is the same as the ratio of the second part to the third coefficient. I.e. $2x^2 + 11x + 12 = 2x^2 + 3x + 8x + 12$, where $2:3 = 8:12$.

Then this ratio, 2:3, gives us one of the factors, $2x + 3$.

The second factor is obtained by:-

(i) dividing the first coefficient of the quadratic (2) by the first coefficient of the factor (2). I.e. $2/2 = 1$, and this is the coefficient of x, and

(ii) dividing the last coefficient in the quadratic (12) by the last coefficient in the factor (3), i.e. $12/3$ gives 4, whence the second factor is $x + 4$

$$\therefore 2x^2 + 11x + 12 = (2x + 3)(x + 4)$$

42.5 FURTHER EXAMPLES

1) $3x^2 + 19x + 20$

We have $3:4 = 15 : 20$. Hence $(3x + 4)$ is a factor. and $3/3 = 1$; $20/4 = 5$, whence $3x^2 + 19x + 20 = (3x + 4)(x + 5)$.

2) $E = 2x^2 + 9x + 9$

Since $2:6 = 3:9$,

$E = (x + 3)(2x + 3)$

3) $6x^2 + 17x + 5 = (2x+5)(3x + 1)$

4) $2x^2 + 3x - 5 = (2x+5)(x - 1)$

5) $2x^2 + 7x + 3 = (x + 3)(2x + 1)$

(6) $3x^2 + 13x - 10 = (x + 5)(3x - 2)$

(7) $9x^2 + 6x - 8 = (3x + 4)(3x - 2)$

(8) $10x^2 + 11x + 3 = (2x + 1)(5x + 3)$

(9) $6x^2 + 25x + 14 = (2x + 7)(3x + 2)$

(10) $10x^2 - 21x + 9 = (2x - 3)(5x - 3)$

A further sub-sutra is useful for checking the answer, namely गुणित समुच्चय : समुच्चय गुणित : Tirthaji translates this as The product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product.

E.g. $(x + 2)(x + 3) = x^2 + 5x + 6$

Putting $x = 1$, we have

$$(1 + 2)(1 + 3) = 1 + 5 + 6 = 12$$

Indeed this sub-sutra finds much wider application in connection with the solution of cubics, biquadratics, etc.

वैदिक गणित अध्ययन

CHAPTER 3

QUADRATIC AND CUBIC EQUATIONS

REFERENCE: The material in this chapter has been freely drawn from Chapter -.5, 11 and 12 of the book Introductory lectures on Vedic mathematics by A.P. Nicholas, J. Pickles and A.K. Williams published by the Vedic Maths Research Group, LONDON.

.3.1 INTRODUCTION

An interesting and unique feature of Vedic Mathematics is that every vedic procedure is directly applicable to various class of problems. The procedure of square rooting once again cuts across the distinction between Arithmetic and Algebraic problem.

.3.2 QUADRATIC EQUATIONS:Page

The facility of the square root procedure comes from its perfect matching of algebraic reasoning to arithmetic use. The unknowns are not notional magnitudes, but are, all along, the decimal digits in which the answer must finally be expressed. There is a directness like that of the carpenter, who when he has to fit a shelf into an alcove, marks its length directly on the timber, and never bothers with the intervening abstractions of feet and inches.

We can think of the calculation of, say $\sqrt{17}$ as a procedure for solving the equation

$$x^2 = 17$$

$$\text{or} \quad x^2 - 17 = 0$$

Indeed, it is a special case of a more general procedure for solving quadratic, cubic, or higher order equations. In every case the vedic method looks to the reality behind the differences in algebraic form, for the solutions, whatever they are, must always be sequences of the nine numbers and zero.

3.2.1 Example 1

Consider the quadratic equation

$$x^2 + 6x - 17 = 0$$

In the traditional system, the solution takes place in two stages. First, algebra provides the standard formula for the unknown x considered as a magnitude:

$$x = \frac{-6 \pm \sqrt{6^2 + 4 \times 17}}{2}$$

Second, arithmetic (or square root tables or a button on a pocket calculator) calculates the square root so that

$$x = \frac{-6 \pm \sqrt{104}}{2}$$

is found as

$$x = 2.099019 \ldots$$

or $$x = -8.099019 \ldots$$

The Vedic procedure unites algebra and arithmetic and finds the solution in one step. We start from an initial chart, writing down the independent term - here 17.000 ... - as in a square root calculation. Taking the first (and here the only) digit pair 17, and matching

$$x^2 + 6x = 17$$

to nearest integers gives x = 2 as the first digit of the answer. When x = 2, $x^2 + 6x = 16$ and so there is a remainder 1. The first digit 2 and the remainder 1 are written down in their places as for a square root calculation:-

$$\frac{17.\;{}_{1}0\;0\;0\;0\;0}{2/}$$

It remains now to set the divisor. We take twice the first digit 2, and add the 6 which is the coefficient of x in the quadratic equation: (2 x 2)+6 = 10. This is just the differential of the quadratic expression

$$\frac{d}{dx}(x^2 + 6x - 17) = 2x + 6$$

evaluated with $x = 2$. The calculation proceeds exactly as for a square root, but with this divisor 10:

$$\begin{array}{ll} 10 & 17.\;{}_{1}0\;0\;{}_{\bar{1}}0\;0\;{}_{2}0\;0\;{}_{\bar{5}}0\;0 \\ \hline & 2.\quad 1\;0\;\bar{1}\;0\;\bar{2}\;0\;\bar{5}\;1 \end{array}$$

giving

$$x = 2.0990195 \;\ldots.$$

The same procedure would apply for the second root of the quadratic, starting with the first digit -8, but it is of course much simpler to use the rule

sum of roots = - coefficient of x = - 6

so that the second root follows immediately as

$$x = -\,8.0990195 \;\ldots.$$

3.3 Algebraic formulation

There is, as before, an algebraic formulation of this procedure. Take a quadratic equation

$$x^2 + Nx = X$$

and suppose that in [illegible] decimal notation

$$X = A.BCDE \;\ldots.$$

and that (for simplicity) the coefficient of x is just a single digit N. Expressing the unknown x as

$$x = a.bcde \;\ldots..$$

or as a power series in $t = 0.1$

$$x = a + bt + ct^2 + dt^3 + \ldots\ldots$$

gives

$$(a+bt+ct^2+..)^2 + N(a + bt + ct^2 + \ldots) = A + Bt + Ct^2 + \ldots$$

This is the quadratic equation considered as a relationship between the digits of the coefficients and the digit-sequence of the solution. To find the digits a, b, c ...., equate coefficients of the powers of t, as in the square root calculation

$$\begin{array}{llll} a^2 + Na & += & A & + \\ (2a + N)bt & + & Bt & + \\ (2a + N)ct^2 + b^2t^2 & + & 6t^2 & + \\ (2a + N)\,dt^3 + 2bct^2 & + & Dt^3 & + \\ & & & \cdots \end{array}$$

and introduce remainders r_A, r_B, etc. where necessary, Note the appearance of the new divisor, $(2a + N)$, as the constant factor on the left hand side.

3.4 FURTHER EXAMPLES

3.4.1 EXAMPLE 2

Take

$$x^2 = x + 1$$

(This is the equation which determines the Golden Ratio ϕ). Writing this as

$$x^2 - x = 1$$

and taking the first digit as 1 would give the first remainder as 1 and a divisor, or differential, of $2x - 1 = 1$.

This divisor is inconveniently small. The difficulty can be sidestepped, either by taking the first two digits together, or by transforming the equation. Multiplying the coefficient of x by 5, and the independent term $5^2 = 25$, gives

$$x^2 - 5x = 25$$

and the new roots are the roots of the original equation multipled by 5. The positive root now begins with the digit 8, the first remainder is $25 - 64 + 40 = 1$, and the differential is $2x - 5 = 11$. Continuing as before

$$\begin{array}{l|l} 11 & 25.\;{}_{1}0\;\;{}_{\bar{1}}0\;\;{}_{0}0\;\;{}_{2}0\;\;{}_{\bar{3}}0\;\;{}_{\bar{1}}0\;\;{}_{0}0\;\;{}_{5}0 \\ \cline{2-2} & 8/.1\;\;\bar{1}\;\;0\;\;2\;\;\bar{3}\;\;0\;\;\bar{1}\;\;4 \end{array}$$

Dividing by 5, or better, doubling and moving the decimal point one place forward, gives the solution of the original equation as

$$x = 1.62\bar{2}04\bar{6}0\bar{2}8\,.. = 1.61803398\,..$$

3.4.2 EXAMPLE 3

What about

$$3x^2 + 2x = 7\,?$$

Dividing through by 3 would make the coefficient of x^2 unity, as in the other examples, but would complicate the other coefficients, introducing recurring decimals. It is better to work with the equation as it stands, where it gives the relationship between the decimal digits of x in the simplest form. The initial chart needs, as before, the first digit 1, the first remainder $7 - 3 \times 1^2 - 2 \times 1 = 2$, and the differential $6x + 2 = 8$:

$$\begin{array}{ll} 8 & 7.\;{}_{2}0\;0\;0\;0\;0\;0\;0 \\ \hline 3 & 1/ \end{array}$$

It is useful also to write down 3, the coefficient of x^2 (representing half the second differential) beneath the first differential 8. The calculation proceeds as before, except that the Dwandwayoga (or duplex) terms are multiplied by this digit 3 at each step. Thus

$$\begin{array}{rl} 20 \div 8 & = 2 \text{ remainder } 4 \\ 40-3\times2^2 = 28,\; 28 \div 8 & = 3 \text{ remainder } 4 \\ 40-3\times(2\times2\times3) = 4,\;\; 4 \div 8 & = 0 \text{ remainder } 4 \\ 40-3\times(2\times2\times0+3^2)=13, 13 \div 8 & = 1 \text{ remainder } 5 \\ 50-3\times(2\times2\times1 + 2\times3\times0)=38, 38 \div 8 & = 4 \text{ remainder } 6 \end{array}$$

and so on. The one-line solution is therefore

$$\begin{array}{ll} 8 & 7.\;{}_{2}0\;{}_{4}0\;{}_{4}0\;{}_{4}0\;{}_{5}0\;{}_{6}0\;{}_{2}0 \\ \hline 3 & 1./\;2\;\;3\;\;0\;\;1\;\;4\;\;\bar{2}\;\;6\ldots \end{array}$$

3.4.3 EXAMPLE 4

$$7x^2 + 15x = 120$$

Here the first digit is 3, the remainder $120 - 7\times3^2 - 15\times3 = 12$, and the differential 14 x 3 + 15 = 57. So

$$\begin{array}{ll} 57 & \underline{120.\ \ 12^0\ \ 6^0\ \ 32^0\ \ 35^0\ \ 39^0\ \ 21^0} \\ 7 & 3/.\ \ \ 2\ \ \ 0\ \ \ 5\ \ \ 3\ \ \ 5\ \ \ \bar{2}\ \ \ldots \end{array}$$

In all these examples, the equation holds within itself the pattern - a cycle of multiplication, subtraction, and division - which relates the successive decimal digits of the unknown x. The solution of the equation consists in displaying the digits which embody the pattern. Consider again Example 3

$$3x^2 + 2x = 7$$

The solution x begins with the digit 1, so let $x = 1 + y$, where y stands for all the digits following. Then

$$\begin{aligned} 3(1 + y)^2 + 2(1 + y) &= 7 \\ 3y^2 + 8y &= 2 \end{aligned}$$

Note that in the new equation for y, the coefficient of y is the differential $6x + 2 = 6 \times 1 + 2 = 8$ of the original equation for x, and the independent term is the original remainder $7 - 3 \times 1^2 - 2 \times 1 = 2$. The new equation can be expressed as

$$y = \frac{2 - 3y^2}{8}$$

which allows y to be written down, digit by digit. The first n digits of y are sufficient to determine the first n digits of y^2, and so of $3y^2$, and these are enough to give the $(n + 1)^{th}$ digit of y. The original quadratic is dissolved into its constituent parts - a sequence of multiplication of the Dwandwayoga by the coefficient 3, subtraction, and division by the differential 8.

4.4.4 EXAMPLE 5

$$2.1x^2 + 2.9x = 15.4$$

The solution here begins with a digits 2, so this gives a differential $4.2x + 2.9 = 11.3$, and a remainder 1.2, or 12 in the first place of decimals. writing $x = 2 + y$ and reasoning as above would give

$$y = \frac{1.2 - 2.1\,y^2}{11.3}$$

and the solution is to be found by a cycle of multiplication of the Dwandwayoga terms by 2.1, subtraction, and division by the differential 11.3. These coefficients are more complicated than in the previous example, and now the intermediate steps can themselves be simplified. Multiplication by 2.1 can be done by'vertically and cross-wise' and division by 11.3 by straight division. To bring all these processes together start with an initial chart

$$\begin{array}{ll|llllllll} 11^3 & & 15. & \overset{-4}{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cline{3-11} 2^1 & & 2./ & & & & & & & & \end{array}$$

showing the remainder and putting the last digits of the differential (11.3) and of the coefficient of x^2(2.1)(on top of the flag) The steps of the calculation are now

$12 \div 11 = 1$ remainder 1

$10 - 2$ (first digit of coefficient 2.1) $\times\ 1^2$ (Duplex)
$- 3$ (flag digit) $\times$ 1 (digit of answer just calculated)
$= 5$
$5 \div 11 = 0$ remainder 5

$50 - 2$ (first digit of 2.1) $\times\ 2 \times 1 \times 0$(Duplex)$-1$ (second digit of 2.1) $\times\ 1^2$(previous Duplex)$-$ 3(flag digit) $\times$ 0 (just calculated) $= 49$, $49 \div 11 = 4$ remainder 5

$50 - 2 \times (2 \times 1 \times 4 + 0^2) - 1 \times (2 \times 1 \times 0) - 3 \times 4 = 22$,
$22 \div 11 = 2$ remainder 0

$0 - 2 \times (2 \times 1 \times 2 + 2 \times 0 \times 4) - 1 \times (2 \times 1 \times 4 + 0^2) - 3 \times 2 = \overline{22}$,

$\overline{22} \div 11 = \bar{2}$ remainder 0

and so on, giving $x = 2.1042\bar{2}$ ...

Note 1

Each Duplex or Dwandwayoga term now turns up twice, once multiplied by the first digit of the coefficient 2.1, and once by the second, it is useful therefore to write it down, on its first appearance, to save recalculating it later. Thus

$$\begin{array}{ll|cccccccc} 11.^3 & & 15. & 12 & 1^0 & 5^0 & 5^0 & 0^0 & 0^0 & 0^0 \\ \cline{3-10} 2.^1 & & 2./ & 1 & 0 & 4 & 2 & \bar{2} & \bar{2} & \bar{3} \\ & & & 0 & 1 & 0 & 8 & 4 & 12 & 12 \end{array}$$

with the Duplex terms written down below in their proper place value positions.

Note 2

Indeed, the place value system can be used more fully, because the $(n+1)^{th}$ Duplex can be calculated for the first time, before the n^{th} Duplex is used again for the second time; tens digits from the $(n+1)^{th}$ Duplex can therefore be carried over, and added into the n^{th} Duplex. while the calculation proceeds. Thus for example,

$$\begin{array}{l|llllllll} 11,^{\bar{5}} & 13. & 12 & 1^0 & 5^0 & 5^0 & 0^0 & 0^0 & \bar{1}^0 \\ \hline 1 & 2./ & 1 & 0 & 4 & 2 & \bar{2} & \bar{2} & \bar{3} \\ & & 0 & 1 & 0 & 8 & 5 & 3 & 2 \end{array}$$

43.5 CUBIC EQUATIONS

43.5.1 EXAMPLE 6

The principles which apply to quadratic equations also apply to cubics, and indeed to higher order equations. Consider

$$x^3 + 13x = 70$$

The first digit of the solution is 3, and with $x = 3 + y$, the equation becomes

$$y^3 + 9y^2 + 40y = 4$$

$$y = \frac{4 - 9y^2 - y^3}{40}$$

Once again knowledge of the first n digits of y on the right hand side of the equation suffices to give the $(n+1)^{th}$ digit on the left hand side.

In calculation, the initial chart can be set up straight away, using the first differential, $3x^2+13 = 3.3^2+13=40$, half the second differential, $\frac{1}{2}(6x) = \frac{1}{2}.6.3 = 9$, one sixth of the third differential = the coefficient of $x^3 = 1$, and the remainder $70 - 3^3 - 13.3 = 4$. Thus

$$\begin{array}{l|llllll} 40 & 70. & 4^0 & 0 & 0 & 0 & 0 & 0 \\ 9 & & & & & & & \\ \hline 1 & 3./ & & & & & & \end{array}$$

In accordance with the structure of the equation for y, the first differential(40) is the divisor at each step, while the dividend is the remainder from the previous step less Duplex digits (corresponding to y^2) multiplied by 9, and less digits corresponding to y^3. One way to calculate the y^3 or 'triplex' term, step by step, is to write the Duplex digits of y^2 beneath the digits of y in the answer, as in Example 5. Vertically-and cross-wise multiplication of the two digit strings for y and for y^2, then gives the required digits of y^3. The steps are

40 ÷ 40 = 1 remainder 0
(and write down the duplex digits 0 and $1^2 = 1$ in the first two places of decimals)

0 (remainder term) - 9 x 1^2(i.e. 9 x duplex) - 1 x 0(triplex product of y and y^2) = - 9,

- 9 ÷ 40 = 0 remainder - 9
(and write down the duplex digit 2 x 1 x 0 in the third decimal place)

$\bar{9}0$ (remainder term) - 9 x 2 x 1 x 0(9 x duplex) - (1 x 1+ 0 x0) (triplex) = $\overline{91}$.

$\overline{91}$ ÷ 40 = $\bar{2}$ remainder $\overline{11}$
(and write down the duplex $2.1.\bar{2} + 0^2 = \bar{4}$ in the fourth decimal place)

$\overline{11}0 - 9 x(2.1.\bar{2} + 0^2$, duplex already calculated as $\bar{4}$)
- (1 x 0 + 0 x 1 + $\bar{2}$ x 0, triplex)
= $\overline{74}$,

$\overline{74}$ ÷ 40 = 2 remainder 6
(and write down duplex $\bar{4}$ in the fifth decimal place)

$60 - 9(2.1.\bar{2} + 2.0.\bar{2}) - (1.\bar{4} + 0.0 + \bar{2}.1 + \bar{2}.0) = 102$,

102 ÷ 40 = 2 remainder 22
(write down duplex 8 in the 6th place)

$220 - 9(2.1.2 + 2.0.\bar{2}+ \bar{2}^2)-(1.\bar{4}+ 0.\bar{4} + \bar{2}.0 + \bar{2}.1 + 2.0) = 154$

154 ÷ 40 = 3 remainder 34
(and write down duplex 14, with the tens digit carried to previous 8)

$$340 - 9\,(2.1.3 + 2.0.\bar{2} + 2.\bar{2}.\bar{2}) + (1.9^{*} + 0.\bar{4} + \bar{2}.\bar{4} + \bar{2}.0 + 2.1 + 3.0)$$
$$= 20\bar{5}$$

$$20\bar{5} \div 40 = 5 \text{ remainder } \bar{5}$$

and so on. This gives $x = 3.10\bar{2}\bar{2}235.. = 3.0978235\ ...$

$$\begin{array}{l|llllllll}
40 & 70. & 4^0 & 0^0 & \bar{9}^0 & \bar{\bar{11}}^0 & 6^0 & 22^0 & 34^0 \\
9 & & & & & & & & \\
\hline
1 & 3./ & 1 & 0 & 2 & \bar{2} & 2 & 3 & 5 \\
 & & 0 & 1 & 0 & 4 & \bar{4} & 2 & 4
\end{array}$$

Note again that in the last step shown the duplex digit 9 (asterisked above) includes the carry figure 1.

43.5.2 EXAMPLE 7

$$x^3 = 70$$

Thus, the calculation of a cube root, is a special case of a cubic equation. With the first digit being 4, the differential terms are $3.4^2 = 48$, $\frac{1}{2}.6.4 = 12$, $1/6\ .\ 6 = 1$, and the first remainder is $70 - 4^3 = 6$. Thus

$$\begin{array}{l|llllllll}
48 & 70. & 6^0 & 12^0 & \;^0 & 23^0 & \bar{8}^0 & \bar{8}^0 & 6^0 \\
12 & & & & & & & & \\
\hline
1 & 4. & 1 & 2 & 1 & 3 & 1 & 5 & 3 \\
 & & 0 & 1 & 4 & 7 & 0 & 1 & 8
\end{array}$$

Note 1

The last duplex term shown, $\bar{8}$, could alternatively be expressed as $\bar{1}2$, and the 1 carried to the left.

Note 2

The vertically and cross-wise multiplication needed to get the triplex is visually easier if the duplex digits remain in a neat row even after adjustment for carried figures. This can be achieved by noting at first only a provisional duplex figure, writing in the fullfigure only when the carry (if any) is established. The last two lines above could alternatively be written

$$\begin{array}{lllllll}
1 & 2 & 1 & 3 & \bar{1} & \bar{5} & 3 \\
0 & 1_4 & 4_6 & 7_0 & 1_1 & 0_2 & 1_3
\end{array}$$

where for example the third duplex $2.1.1 + 2^2 = 6$ is written, at first, as a subscript and is confirmed as 7 only when the carry from the next duplex $2.1.3 + 2.2.1 = 10$ is known.

3.5.3 EXAMPLE 8

$$x^3 - 7x^2 + 20x = 36$$

With the first digit 4, the differential terms are $3.4^2 - 14.4 + 20 = 12$, $\frac{1}{2}(6.4 - 14) = 5$, $1/6\ .6 = 1$, and the first remainder is $36 - 4^3 + 7.4^2 - 20.4 = 4$. So

$$\begin{array}{llllllll}
12 & 36. & 4^0 & 4^0 & 4^0 & 14^0 & \overline{12}^0 & \bar{8}^0 \\
5 & & & & & & & \\
\hline
 & 4./3 & \bar{1} & 5 & 0 & \bar{5} & \bar{3} & \\
 & & 1_{\bar{1}} & \bar{1}_{\bar{6}} & \bar{3}_1 & 0_0 & 0_{\bar{5}} & \bar{6}_2
\end{array}$$

and $x = 4.3\bar{1}50\bar{5}\bar{3}.. = 4.294947...$

Without the carries in the duplex terms, the same calculation would be written

$$\begin{array}{lllllllll}
12 & 36. & 4^0 & 4^0 & 5^0 & \overline{17}^0 & \bar{8}^0 & 25^0 \\
5 & & & & & & & \\
\hline
1 & 4/ & 3 & 0 & \bar{5} & \bar{1} & \bar{5} & \\
 & & 0 & 9 & 0 & \overline{36} & \bar{5} & 55
\end{array}$$

and the large duplex terms now make the calculation of further decimal digits difficult.

3.5.4 EXAMPLE 9

$$x^3 - 7x^2 + 20x = 37$$

With the same first digit and differentials as in Example 8, but with a remainder now of 5, the calculation begins

$$\begin{array}{llllll}
12 & 37. & 5^0 & 2^0 & \bar{4}^0 & 0 \\
5 & & & & & \\
\hline
1 & 4/. & 4 & \bar{5} & & \\
 & & 1_6 & 2 & &
\end{array}$$

but the calculation is already awkward because the Dwandvayoga corrections are large compared with the divisor 12. One remedy is to work with the transformed equation

$$z^3 - 2.7.z^2 + 2^2.20z = 2^3.37$$

or

$$z^3 - 14z^2 + 80z = 296$$

with $z = 2x$. Now the first digit is $z = 9$, the differential terms are $3.9^2 - 28.9 + 80 = 71$, $\frac{1}{2}.(6.9 - 28) = 13$, and $1/6\ .6 = 1$, and the remainder is $296 - 9^3 + 14.9^2 - 30.9 = \overline{19}$. The solution for z is given by

$$\begin{array}{c|cccccccc}
71 & 296. & \overline{19}^0 & 23^0 & \overline{26}^0 & 30^0 & \overline{44}^0 & 9^0 \\
13 & & & & & & & \\
\hline
1 & 9/. & \bar{3} & 2 & \bar{2} & 2 & \bar{3} & \bar{4} \\
 & & 1_{\bar{1}} & \bar{2}_{2} & 0_{\bar{4}} & \bar{6}_{0} & 3_{0} & 0_{4}
\end{array}$$

with $z = 9.\bar{3}2\overline{22}\bar{3}\bar{4}$ $x = 4.36\bar{1}1\overline{17} = 4.359083 \ldots$

3.5.5 EXAMPLE 10

The above solution requires, apparently, a division by 71. But in the vedic system 71 is just a 7 and a 1, and the calculation can be so framed as to need only division by 7. For example, in the working above the division of 19^0 (in the tenths column) by 71 can be treated as a division of 19 by 7, with an appropriate adjustement for a flag digit 1. The second differential term 13 similarly becomes a 1 with 3 on the flag and the third differential term 1 becomes 0^1. The full solution is now expedited by writing the triplex digits (y^3) as well as the duplex digits (y^2) below the decimal digits (y) of the solution,

Thus

$$\begin{array}{c|cccccccccc}
7^1 & 2 & 9 & \overset{6}{\overline{19}}. & 2^0 & \bar{3}^0 & 0 & 0 & 0 & 0 \\
1^3 & & & & & & & & & \\
\hline
0^1 & & 9./ & \bar{3} & 2 & & & & & \\
 & & & 1_{\bar{1}} & \bar{2}_{\bar{2}} & & & & & \\
 & & & 0_{\bar{3}} & \bar{2}_{\bar{2}} & & & & &
\end{array}$$

and at this stage the net dividend is

$\bar{3}0$ (remainder term)

- 1 (flag digit of 71) x 2 (digit of solution in second decimal place)

- 1 (first digit of coefficient 13) x $\overline{12}$(current duplex)

- 3 (flag digit of 13) x $\bar{2}$ (duplex digit in second decimal place)

- 0 (first digit of 01) x current triplex

- 1 (flag digit of 01) x $\bar{2}$(triplex digit in second decimal place, just obtained as $\bar{3}$ (carried from left) + tens digit of current vertically and crosswise product $(y \times y^2)1\bar{2} = \bar{3} \times \bar{2} + 1 \times 2 = \overline{12}$ and $\overline{12} \div 7 = \bar{2}$ remainder 2

Continuing in the same way

$$\begin{array}{llllllllllll}
7^1 & 2 & 9 & \overset{6}{\overline{19}}. & 2^0 & \bar{3}^0 & 2^0 & \bar{5}^0 & 1^0 & 1^0 & \bar{1}^0 \\
1^3 & & & & & & & & & & \\
\hline
0^1 & & 9./ & \bar{3} & 2 & \bar{2} & 2 & \bar{3} & \bar{4} & 1 & \\
 & & & 1_{\bar{1}} & \bar{2}_{\bar{2}} & 0_{\bar{4}} & \bar{6}_0 & 3_0 & 0_4 & 3_4 & \\
 & & & 0_{\bar{3}} & \bar{2}_{\bar{2}} & \bar{3}_4 & 6_4 & 1_2 & 4_0 & \bar{2}_2 &
\end{array}$$

giving $z = 9.\bar{3}2\bar{2}\bar{3}\bar{4}1 \ldots$ or $x = 4.3590830\ldots$

43.5.6 EXAMPLE 11

$$7x^3 + 13\,x = 70$$

The first digit is 2, giving differential terms 97, 42, and 7, and a first remainder $70 - 7.2^3 - 13.2 = \overline{12}$. So

$$\begin{array}{llllllll}
97 & 70. & \overline{12}^0 & \overline{23}^0 & 19^0 & 49^0 & 49^0 & 0 \\
42 & & & & & & & \\
\hline
7 & 2/. & \bar{1} & \bar{3} & \bar{1} & 0 & & \\
 & & 0_1 & 2_{\bar{4}} & \bar{3}_1 & 2_{\bar{4}} & &
\end{array}$$

For the next step the net dividend will be

490 (remainder term, or gross dividend)

$-$ 42 x $(2.\bar{1}.0 + 2.\bar{3}.\bar{1}$, current duplex)

$-$ 7 x $(\bar{1}.2 + \bar{3}.\bar{3} + \bar{1}.2 + 0.0$, current triplex)

= 203, and 203 ÷ 97 = 2 remainder 9.

CHAPTER-4

TRIGNOMETRICAL HYPERBOLIC FUNCTION' AND THEIR INVERSE

REFERENCE: - The material has been freely drawn from Chapter No.9 and 10 of the book. Applications of the Vedic Mathematics Sutra URDHAVA TIRYAG BHAYAM

VERTICALLY AND CROSS WISE BY A.P. NICHOLAS, K.R. WILLIAMS AND J. PICKLES published by the Vedic Maths Research Group LONDON.

4.1 SINE, COSINE AND INVERSE TANGENT:

By using the left-to-right methods of calculation described in chapter 1, and the series expansions for sinx, cosx and inverse tanx, these functions can be evaluated for any angle. This is the subject of the present chapter, and in the next chapter we show how these same expansions can be used in reverse to evaluate inverse sines, inverse cosines and tangents.

Cosine: The series expansion for cosx is:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \quad \text{where } x \text{ is in radians}$$

If we write this as $\cos x = 1 + A + B + C + D + \ldots$

we see that $B = \frac{A^2}{6}$, $C = \frac{AB}{15}$, $D = \frac{AC}{28}$ etc.

This means that once we have obtained the value of $\frac{x^2}{2}$ the next term is obtained by squaring this and dividing by 6, the next by multiplying the two previous terms and dividing by 15, and so on.

4.1.1 ILLUSTRATION 1 : Cosine 0.2 radians

We set up a chart as shown:

$1 - x^2/2$	1.0	$\bar{2}$									row 1
$x^4/24$ 6	.0	0	0	0	6	6	6	6	6	6	2
$-x^6/720$ 15						$\bar{1}$	$\bar{1}_3$	1_3	1_3	1_3	3
cos 0.2 =	1.0	$\bar{2}$	0	0	6	6	5	7	7	7	4

We require the figures in row 4, and these are the sum of the figures in rows 1, 2, 3, as can be seen by looking at the left-hand side. As explained above the divisors in rows 2,3 are 6, 15 and it is useful to have these written down.

We put 1 in row 1, followed by $\frac{-0.2^2}{2}$ ie. by $0.0\bar{2}$.

Then square the decimal part of row 1 and divide by 6 to obtain row 2:

$$0.02^2 = 0.0004, \quad 0.0004 \div 6 = 0.00006.$$

Then multiply rows 1,2 and divide by 15:

$\bar{2}.6 = \overline{12}$, $\overline{12} \div 15 = \bar{1}$ r3, put down $\bar{1}_3$.

$\bar{2}.6 = \overline{12}$, $\overline{12}$ + carried 3 (as 30) = 18, 18 ÷ 15 = 1 r3,etc

We then add the columns up. In fact the next term in the series comes in at the 10th decimal place, so that the last figure should be 8.

$\therefore$ cos 0.2 = 0.980066578 to 9 D.P

4.1.2 ILLUSTRATION 2: Cosine 0.3 radians

$1 - x^2/2$		1.0	$\bar{4}$	$\bar{5}$						1
$x^4/24$	6				$3_{\bar{2}}$	3_2	7_3	5		2
$-x^6/720$	15						$\bar{1}_3$	0_3	$\bar{1}_2$	3
cos 0.3 =		1.0	$\bar{4}$	$\bar{5}$	3	3	6	5	$\bar{1}$	4

$\therefore$ cos 0.3 = 0.9 5 5 3 3 6 4 9 to 8 D.P.

Row 1 : $\frac{-0.3^2}{2} = 0.0\overline{45}$.

Row 2 : we need to square $\overline{45}$, so we find the duplexes in turn, and divide each of them by 6: $D(\bar{4}) = 16$, $16 \div 6 = 3$ r$\bar{2}$, put $3_{\bar{2}}$.

$D(\overline{45}) = 40$, 40 + carried $\bar{2}$(as $\overline{20}$)=20, 20 ÷ 6=3r2 put3_2.

$D(\bar{5}) = 25$, 25 + 20 = 45, 45 ÷ 6 = 7_3.

The carried 3 is 30 in the next column, so 30 ÷ 6 = 5.

Row 3: we multiply rows 1,2: $\mathrm{CP}\left[\begin{smallmatrix}\bar{4}\\3\end{smallmatrix}\right] = \overline{12}$, $\overline{12} \div 15 = \bar{1}_3$.

$\mathrm{CP}\left[\begin{smallmatrix}\overline{45}\\33\end{smallmatrix}\right] = \overline{27}$, $\overline{27}$ + 30 = 3, 3 ÷ 15 = 0_3.

$\mathrm{CP}\left[\begin{smallmatrix}\overline{45}\\37\end{smallmatrix}\right] = \overline{43}$, $\overline{43}$+ 30 = $\overline{13}$, $\overline{13} \div 15 = \bar{1}_2$.

4.1.3 SINE :- The series expansion for sinx is:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots.$$

Writing this as sinx = A - B + C - D + ...

we see that $B = \frac{Ax^2}{6}$, $C = \frac{Bx^2}{20}$, $D = \frac{Cx^2}{42}$ etc.

This means that we can obtain x^2 and then evaluate each term by multiplying by x^2 and dividing by 6, 20, 42 etc.

4.1.4 ILLUSTRATION 3: Sin 0.2

(x^2)			.	0	4)								0
x		0	.	2									1
$-x^3/3!$	6		:	$\dot{0}$	0	$\bar{1}_2$	$\bar{3}_2$	$\bar{3}_2$	$\bar{3}_2$	$\bar{3}_2$	$\bar{3}_2$	$\bar{3}_2$	2
$x^5/5!$	2								2	6	6	6	3
$-x^7/7!$	42											$\bar{2}_{20}$	4
sin 0.2 =		0	.	2	0	$\bar{1}$	$\bar{3}$	$\bar{3}$	$\bar{1}$	3	3	1	5

First we find that $x^2 = 0.04$.

For row 2 we multiply rows 0,1 and divide by 6:

4.2 = 8, 8 ÷ 6 = 1 r2, put down 1_2 (we will change the signs of rows 2,4 later).

Take the carried 2, as 20, 20 ÷ 6 = 3_2 etc.

For row 3 multiply rows 0,2 and divide by 2:

4.1 = 4, 4 ÷ 2 = 2. 4.3 = 12, 12 ÷ 2 = 6, etc.

For row 4 multiply rows 0.3 and divide by 42:

4.26 = 104, 104 ÷ 42 = 2_{20}.

Then change the signs of rows 2,4 and add the columns up.

$\therefore$ sin 0.2 = 0.19866933 to 8 D.P.

4.1.5 ILLUSTRATION 4 : Sin 0.3

$(x^2$			.	1	$\bar{1})$						
x		0	.	3							
$-x^3/3!$	6		.	0	0	$\bar{4}_3$	$\bar{5}$				
$x^5/5!$	2							2	0_1	2	5
$-x^7/7!$	42										$\bar{4}$
sin 0.3 =		0	.	3	0	$\bar{4}$	$\bar{5}$	2	0	2	1
sin 0.3 =		0	.	2	9	5	5	2	0	2	

4.1.6 INVERSE TANGENT:

The series is : $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots.$

4.1.7 ILLUSTRATION 5: Tan^{-1} 0.2

x	0	.	2						
$-x^3/3$		.	0	0	$\bar{3}$	3	3	3	3
$x^5/5$							6	4	
$-x^7/7$								$\bar{2}$	2
$\tan^{-1}0.2=$	0	.	2	0	$\bar{3}$	3	9	5	5

Here it is very easy to evaluate the terms and add them up.

4.1.8 ILLUSTRATION 6: Tan^{-1} 0.342

x	0	.3	4	2		1
$-x^2/3$			$\bar{1}$	$\bar{3}_1$	$\bar{3}_2$	2
$+x^5/5$				$1_{\bar{2}}$	1_1	3
$\tan^{-1}0.342=$	0	.3	3	0	$\bar{4}$	4
$(x^2$		$.1_1$	$\bar{2}_{\bar{6}}$	$\bar{3}_{\bar{2}}$	)	5
$(x^4$		1	4		)	6

We first find x^2 in row 5.

For row 2 we multiply rows 1,5 and divide by 3:

$CP\begin{bmatrix}3\\1\end{bmatrix} = 3,\ 3 \div 3 = 1.$ $CP\begin{bmatrix}34\\12\end{bmatrix} = 10,\ 10 \div 3 = 3_1.$

$CP\begin{bmatrix}342\\12\bar{3}\end{bmatrix} = 1,\ 1 + 10 = 11,\ 11 \div 3 = 3_2.$

For row 3, square row 5 (row 6), multiply by row 1, and divide by 5:

$D(1) = 1$, $CP\left[{3 \atop 1}\right] = 3$, $3 \div 5 = 1_{\bar{2}}$.

$D(12) = 4$, $CP\left[{34 \atop 14}\right] = 16$, $16 + \overline{20} = \bar{4}$, $\bar{4} \div 5 = \bar{1}_1$

Change the signs of the figures in row 2, and add up the columns.

$\therefore \tan^{-1} 0.342 = 0.330.$

4.2 HYPERBOLIC FUNCTIONS:

These can also be expressed simply in series form, and so can be evaluated like the previous examples.

4.2.1 ILLUSTRATION 7: Sinh 0.41

We have $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \; ....$

$\therefore$ x		0	.	4	1				
$x^3/3!$	6				1_2	$2_{\bar{2}}$	$\bar{5}_{\bar{1}}$	$\bar{1}_{\bar{2}}$	$\bar{3}_{\bar{1}}$
$x^5/5!$	2						1_0	0_1	$\bar{4}_0$
$\therefore$ sinh 0.41 =		0	.	4	2	2	$\bar{4}$	$\bar{1}$	$\bar{7}$

$\therefore$ sinh 0.41 = 0.42158 to 5 D.P.

Similarly we can evaluate cosh x and $\tanh^{-1} x$ using:

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \; ....$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \; ....$$

However, hyperbolic functions are not periodic like the circular functions, so that if x is large a lot of work would be involved. But in such cases we can use the methods described in chapter 7 of the reference to evaluate the following alternative expressions for sinh, cosh and tanh:

$$\sinh x = \tfrac{1}{2}(e^{-x} - e^{x}) \qquad \cosh x = \tfrac{1}{2}(e^{-x} + e^{x})$$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

4.3 INVERSE OF SINE, COSINE AND TANGENT:

The ability to make calculations from left to right means that we can obtain the most significant digit of an answer first, then the next most significant and so on. In this chapter and the next we show how the digits of an answer, can be used to obtain the next digit.

4.3.1 INVERSE OF SINE:

$$\text{Since } \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \ldots$$

$$\therefore \quad x = \sin x + \frac{x^3}{3!} + \frac{x^5}{5!} + \quad \ldots.$$

4.3.2 ILLUSTRATION 8:

Find $\sin^{-1} 0.4$ to 4 D.P.

Here we are given $\sin x = 0.4$, and we have to find x

$$\therefore \quad x = 0.4 + \frac{x^3}{3!} - \frac{x^5}{5!} + \quad \ldots.$$

We therefore set up a chart to evaluate the terms on the RHS of this equation:

sin x		0	.	4					row 1
$x^3/3!$	6				1_2	1_4	$6_{\bar{1}}$	1_2	2
$-x^5/5!$	2						$\bar{1}$	$0_{\bar{1}}$	3
x =		0	.	4	1	1	5	1	4
(x^2 =			.	$2_{\bar{4}}$	$\bar{3}_{\bar{2}}$	$\bar{1}_{\bar{1}}$	3_2	)	5

The answer appears in row 4, and rows 2, 3 and 5 are formed digit by digit as x is found. We therefore work vertically (rather than horizontally as in the last chapter) and as each digit of x is found it is used to obtain the next one.

First bring 0.4 down into the answer. Then for row 5, $D(4) = 16 = 2\bar{4}$, put $2_{\bar{4}}$.

Next, for row 2, multiply rows 4,5 and divide by 6:

$CP \left[\begin{smallmatrix}4\\2\end{smallmatrix}\right] = 8$, $8 \div 6 = 1_2$.

Bring down 1 into the answer; then for row 5, $D(41) = 8$, $8 + \overline{40} = \overline{32}$, put $\bar{3}_{\bar{2}}$

Row 2 again: $CP\left[\begin{smallmatrix}4 & 1\\2 & \bar{3}\end{smallmatrix}\right] = \overline{10}$, $\overline{10} + 20 = 10$, $10 \div 6 = 1_4$.

Bring down 1 into the answer, Row 5: $D(411) = 9$, $9 + \overline{20} = \bar{1}_{\bar{1}}$

Row 2: $CP\left[\begin{smallmatrix}4 & 1 & 1\\2 & \bar{3} & \bar{1}\end{smallmatrix}\right] = \bar{5}$, $\bar{5} + 40 = 35$, $35 \div 6 = 6_{\bar{1}}$

Row 3: we multiply rows 2, 5 and divide by 2: $CP\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right] = 2$, $2 \div \underline{2} = 1$, put $\bar{1}$
as this term is negative.

Bring down 5 into the answer

Row 5: $D(4115) = 42$, $42 + \overline{10} = 3_2$.

Row 2: $CP\left[\begin{smallmatrix}4115\\2\overline{31}3\end{smallmatrix}\right] = 18$, $18 + \overline{10} = 8$, $8 \div 6 = 1_2$.

Row 3: $CP\left[\begin{smallmatrix}1 & 1\\2 & \bar{3}\end{smallmatrix}\right] = \bar{1}$, $\bar{1} \div 0_{\bar{1}}$

Bring down 1 into the answer.

$\therefore$ $x = 0.4115$ to 4 D.P.

(the other solution between 0 and 2π is $\pi - 0.4115$)

4.3.3 INVERSE OF COSINE:

ILLUSTRATION 9:

$\cos^{-1} 0.8$

Since $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots\ldots$

$$\therefore x^2 = 2(1 - \cos x) + \frac{x^4}{12} - \frac{x^6}{360} \ldots$$

And if we let $y = x^2$ we have $y = 2(1 - \cos x) + \frac{y^2}{12} - \frac{y^3}{360} + \ldots$

so that we can evaluate y and take its square root to find x.

$\therefore$ if $\cos x = 0.8$, $y = 2(1 - 0.8) + \frac{y^2}{12} - \frac{y^3}{360} + \ldots\ldots$

$2(1-\cos x)$		$0\ .\ 4$	1
$y^2/12$	12	$1_4\ 4_0\ 3_{\bar{3}}\ \bar{\bar{1}}_{\bar{2}}$	2
$-y^3/360$	3	$\bar{2}_{\bar{2}}\ 0_{\bar{3}}$	3
$y = x^2 =$		$0\ .\ 4\ 1_5\ 4_6\ 1_9\ \bar{\bar{1}}_5$	4
$x =$		$0.\ 6\ 4\ 3\ 5$	5

First we bring 0.4 down into row 4.

Row 2: $D(4) = 16$, $16 \div 12 = 1_4$. Bring down 1 into row 4.

Row 2: $D(41) = 8$, $8 + 40 = 48$, $48 \div 12 = 4_0$. Bring down 4 into row 4.

Row 2: $D(414) = 33$, $33 + 0 = 33$, $33 \div 12 = 3_{\bar{3}}$.

Row 3: multiply rows 2, 4 and divide by 3: CP$\begin{bmatrix}1\\4\end{bmatrix}$= 4, 4 ÷ 3=2r$\bar{2}$, put $\bar{2}_{\bar{2}}$ as this term is negative.

Bring down 1 into row 4.

Row 2: D(4141) = 16, 16 + $\overline{30}$ = $\overline{14}$, $\overline{14}$ ÷ 12 = $\bar{1}_{\bar{2}}$

Row 3: CP$\begin{bmatrix}1 & 4\\4 & 1\end{bmatrix}$ = 17, 17 + $\overline{20}$ = $\bar{3}$, $\bar{3}$ ÷ 3 = $0_{\bar{3}}$.

Bring $\bar{1}$ into row 4.

We then find the square root of row 4 using the usual Vedic method as described on [illegible].

4.3.4 ILLUSTRATION 10:

Cos^{-1} 0.123

Since this angle is close to 90°, we can find $\sin^{-1}0.123$ and subtract the result from $\pi/2$:

sin x		0 . 1 2 3
$x^3/3!$	6	$3_{\bar{1}}\ 1_{\bar{1}}\ 2_0$
x =		0 . 1 2 3 3 1 2

($x^2 = .00_1\ 1_4\ 5_0\ 2_{\bar{2}}0_3$)

$\therefore \cos^{-1}0.123 = \pi/2 - 0.123312 = 1.447484$

4.3.5 INVERSE OF TANGENT

ILLUSTRATION 11: Tan^{-1} 0.3

We have $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$

$$\therefore \quad x = \tan^{-1}x + \frac{x^3}{3} - \frac{x^5}{5} + \dots$$

$\tan^{-1}x$		0 . 3	1
$x^3/3$	3	$1_0\ 0_1\ \bar{1}_0$	2
$-x^5/5$	5	$\bar{1}_{\bar{2}}\ \overset{+}{4}_0$	3
x =		0 . 3 1 $\bar{1}$ 3	4
(x^2 =		$.1_{\bar{1}}\ 0_{\bar{4}}\ \bar{4}_{\bar{5}}$)	5

Put 0.3 in the answer. Row 5: D(3) = $1_{\bar{1}}$.

Row 2: $CP\begin{bmatrix}3\\1\end{bmatrix} = 3,\ 3 \div 3 = 1_0$. Put 1 in the answer. Row 2: $D(31) = 6$, $6 + \overline{10} = 4$, put $0_{\bar{4}}$.

Row 2: $CP\begin{bmatrix}3 & 1\\1 & 0\end{bmatrix} = 1,\ 1 + 0 = 1,\ 1 \div 3 = 0_1$.

Row 3: multiply rows 2,5, multiply by 3 and divide by 5: $CP\begin{bmatrix}1\\1\end{bmatrix} = 1,\ 1.3 = 3,\ 3 \div 5 = 1_{\bar{2}}$.

Put $\bar{1}$ in the answer

Row 5: $D(31\bar{1}) = \bar{5},\ \bar{5} + \overline{40} = \bar{4}_{\bar{5}}$. Row 2: $CP\begin{bmatrix}3 & 1 & \bar{1}\\1 & 0 & 4\end{bmatrix} = \overline{13},\ \overline{13}+10 = \bar{3}$, $\bar{3} \div 3 = \bar{1}_0$.

Row 3: $CP\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix} = 0,\ 0.3 = 0,\ 0 + \overline{20} = \overline{20},\ \overline{20} \div 5 = \bar{4}_0$. Put 3 in the answer.

Note in this last line that we multiply by 3 and add the carried figure before dividing by 5.

4.4 HYPERBOLIC FUNCTION:

We can similarly find inverse hyperbolic sines, inverse hyperbolic cosines and tangents, using the series expansions given in the last chapter. The method is just like that of the previous examples of this chapter, so we give just one example, without explanations:

4.4.1 ILLUSTRATION 12:

$\mathrm{Sinh}^{-1} 0.5432$

$$\therefore\quad 0.5432 = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots.$$

$$\therefore\quad x = 0.5432 - \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots.$$

$$\therefore\quad \begin{array}{l|ccccccc}
\sinh x & 0 & . & 5 & 4 & 3 & 2 & \\
-\,x^3/3!\ \ 6 & & & & \bar{2}_3 & \bar{3}_3 & \bar{4}_0 & \overset{+}{3}_{\bar{2}} \\
-\,x^5/5!\ \ 2 & & & & & & \bar{3}_0 & \bar{1}_{\bar{1}} \\
\hline
x\ \ = & 0 & . & 5 & 2 & 0 & \bar{5} & 2 \\
\hline
(x^2\ \ = & & . & 3_{\bar{5}} & \bar{3}_0 & 0_4 & \bar{1}_0 &)
\end{array}$$

$$\therefore\quad \sinh^{-1} 0.5432 = 0.5195 \text{ to 4 D.P.}$$

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

T4

TUTORIAL-4

वैदिक गणित अध्ययन

Inverse Sine, Cosine, Tan.

Evaluate the following using Vedic method

1. $\sin^{-1} 0.125$
2. $\sin^{-1} 0.283$
3. $\sin^{-1} 0.111$
4. $\sin^{-1} 0.218$
5. $\sin^{-1} 0.212$
6. $\sin^{-1} 0.101$
7. $\sin^{-1} 0.923$
8. $\sin^{-1} 0.841$
9. $\tan^{-1} 0.246$
10. $\tan^{-1} 1.241$
11. $\tan^{-1} 1.269$
12. $\tan^{-1} 0.547$
13. $\tan^{-1} 0.125$
14. $\tan^{-1} 0.201$
15. $\cos^{-1} 0.210$
16. $\cos^{-1} 0.012$
17. $\cos^{-1} 0.054$
18. $\cos^{-1} 0.602$
19. $\cos^{-1} 0.321$
20. $\cos^{-1} 0.105$
21. $\cos^{-1} 0.540$

वेदात् सर्वं प्रसिद्धयति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

वैदिक गणित अध्ययन

CHAPTER 5

POLYNOMIAL EQUATIONS

The Reference is same as that of the Previous Chapter.

5.1 INTRODUCTION

In all these examples we have effectively been solving polynomial equations because we have discarded all the terms of the series after a certain point. In the example above we solved a quintic equation. We can generally solve polynomial equations in this way provided that the root we are looking for lies between 1 and -1. If it does not, then we can transform the equation, using the methods described in Chapter 8 of the reference, so that the root is in this range, and then solve it. The following are a few examples of this, first without and then with change of roots.

5.1.1 ILLUSTRATION 1

$$x^2 + 5x - 1 = 0$$

$\therefore \quad x = \frac{1-x^2}{5}$ we can see that this equation has a root at about $x = 0.2$. We therefore solve

$$x = 0.2 - \frac{x^2}{5}$$

$$\begin{array}{lllllll}
0.2 & 0\,.\,2 & & & & \\
-x^2/5 & & \bar{1}_{\bar{1}} & 3_1 & \bar{4}_3 & \bar{2}_{\bar{2}} \\
\hline
x = & 0\,.\,2 & \bar{1} & 3 & \bar{4} & \bar{2} \\
\hline
\end{array}$$

Bring down 2 into the answer. Then $D(2) = 4$, $4 \div 5 = 1 \text{ r} \bar{1}$, put $1_{\bar{1}}$. Bring down $\bar{1}$. Then $D(2\bar{1}) = \bar{4}$, $\bar{4} + \bar{1}\bar{0} = \bar{1}\bar{4}$, $\bar{1}\bar{4} \div 5 = \bar{3}\text{r}1$, put 3_1. And so on.

$\therefore \quad x = 0.19258$ to 5 D.P.

5.1.2 ILLUSTRATION 2:

$$2x^3 - 4x^2 - 10x + 3 = 0$$

If we rearrange this equation to get $x = 0.3 - \frac{2x^2}{5} + \frac{x^3}{5}$

We see that there is a root at about $x = 0.3$. We therefore proceed as before:

0.3	0	.	3					1
$-2x^2/5$				$\bar{3}_3$	$0_{\bar{6}}$	0_6	$\bar{5}_{\bar{1}}$	2
$x^3/5$					4_1	$1_{\bar{1}}$	$2_{\bar{2}}$	3
x =	0	.	3	$\bar{3}$	4	1	$\bar{3}$	4

Bring down 0.3 into the answer.

We need to square x, multiply by 2, divide by 5 and change the sign:

$D(3) = 9$, $9.2 = 18$, $18 \div 5 = 3\ r3$, put $\bar{3}_3$

Bring down $\bar{3}$

$D(3\bar{3})$ $\overline{18}$, $\overline{18}.2 = \overline{36}$, $\overline{36} + 30 = \bar{6}$, $\bar{6} \div 5 = 0_{\bar{6}}$.

Row 3: We change the sign of the figures in row 2, multiply by x (i.e. by row 4), and divide by 2: $CP\{\substack{3\\3}\} = 9$, $9 \div 2 = 4_1$.
Etc.

Alternatively, we can evaluate $x = 0.3 - 0.4x^2 + 0.2\,x^3$

0.3	0	.	3						1
$-0.4x^2$				$\bar{3}_6$	$0_{\overline{12}}$	0_{12}	$\bar{5}_{\bar{2}}$	$\overset{+}{4}_{\overline{12}}$	2
$0.2x^3$					4_1	$1_{\bar{1}}$	$2_{\bar{2}}$	$\bar{3}_4$	3
x	0	.	3	$\bar{3}$	4	1	$\bar{3}$	1	4

Bring down 0.3 into the answer.

$D(3) = 9$, $9.4 = 36$, put $\bar{3}_6$

Bring down $\bar{3}$

$D(3\bar{3}) = \overline{18}$, $\overline{18}.4 = \overline{72}$, $\overline{72} + 60 = \overline{12}$ put $0_{\overline{12}}$. Etc.

Row 3 is found just as before.

$\therefore$ $x = 0.274071$ to 6 D.P.

5.1.3 ILLUSTRATION 3

$$7x^2 - 50x + 8 = 0$$

This can be rearranged to $x = 0.16 + \frac{7x^2}{50}$ so that there is a root near $x = 0,2$.

$$\therefore\quad \begin{array}{r|l} 0.16 & 0\ .\ 2\ \bar{4} \\ 7x^2/50 & \qquad {}^{0}{}_{28}\ {}^{3}{}_{18}\ {}^{7}{}_{26}\ {}^{5}{}_{14} \\ \hline x = & 0\ .\ 2\ \bar{4}\ \ 3\ \ 7\ \ 5 \\ \hline \end{array}$$

The other root is not less than 1, but by transforming the equation to another with reciprocal roots, we will be able to find it:

The transformed equation is $8x^2 - 50x + 7 = 0$

$$\therefore\quad x = \frac{7 + 8x^2}{50} = 0.14 + \frac{4x^2}{25}$$

$$\begin{array}{r|l} 0.14 & 0\ .\ 1\ \ 4 \\ 4x^2/25 & \qquad\qquad 3_{\bar{3}}\ 2_{8}\ 8_{\bar{8}} \\ \hline & 0\ .\ 1\ \ 4\ \ 3\ \ 2\ \ 8 \\ \hline \end{array}$$

We then find the reciprocal of this using the Vedic method of straight division.

$$\begin{array}{l|l} 1\ 4\ {}^{3\ 3\ \bar{2}} & 1\ 0_{\bar{4}}0_{\bar{1}}.\ 0_{\bar{4}}0_{\bar{1}}0_{4}0 \\ \hline & 1\ \bar{3}\ .\ 0\ \bar{2}\ \bar{1} \\ \hline \end{array}$$

The two roots are therefore $x = 0.16375, 6.979$.

We could have used other transformations instead of the one used here, to get the second root- we could, for example, have reduced the roots by 7.

Also since the sum of the roots of a quadratic equation is minus the coefficient of x divided by the coefficient of x^2, the second root can be easily obtained from the first.

वैदिक विज्ञान संस्थान
आध्यात्मिक अध्ययन संस्था (आसस)

INSTITUTE OF VEDIC SCIENCES
SPIRITUAL STUDY GROUP (SSG)
41/2, Amod Kunj
University Campus
ROORKEE - 247 667 (INDIA)

वैदिक गणित अध्ययन

CHAPTER 6

TRANSCENDENTAL EQUATION

(KEPLER'S EQUATION)

REFERENCE:- Chapter 11 of the book Referred in Chapter 4.

6.1 INTRODUCTION:

The methods described in the last chapter can be applied to the solution of transcendental equations.

6.1.1 ILLUSTRATION 1:

$$x + \sin x = 1$$

$$\therefore x + x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = 1$$

$$\therefore x = 0.5 + \frac{x^3}{12} - \frac{x^5}{240} + \frac{x^7}{2.7!} \dots\dots$$

0.5		0	.	5				
$x^3/12$	12				1_3	1_1	1_2	2_2
$-x^5/240$	2						$\bar{1}_1$	$\bar{5}_{\bar{1}}$
x =		0	.	5	1	1	0	$\bar{3}$
$(x^2 =$			.	3_5	$\bar{4}_0$	1	1_2	$\bar{1}_1$)

The method is quick and efficient if x is small, and can be extended to any number of decimal places.

6.2 KEPLER'S EQUATION

And equation of great importance in positiona astronomy. This equation is

$$M = E - e \sin E$$

where M, E are the mean and eccentric anomalies respectively of a planet in its orbit, and e is the eccentricity of th orbit.

We will take M = 0.30303 radians and e = 0.016722 - which is the eccentricity of the Earth's orbit. We want t find E.

6.2.1 ILLUSTRATION 2:

$$0.30303 = E - 0.16722 \sin E$$

$$\therefore \quad E = 0.30303 + 0.016722\left(E - \frac{E^3}{\underline{3|}} + \frac{E^5}{\underline{5|}} - \cdots\right.$$

M	0	.	3	0	3	0	3	0	0	1
eE				$1_{\bar{4}}$	$\bar{5}_3$	$2_{\bar{6}}$	$\bar{5}_1$	2_1	$1_{\bar{3}}$	2
$-eE^3/\underline{3\rfloor}$ 6						$\bar{1}_{\bar{1}}$	$2_{\bar{1}}$	$\bar{1}_3$	$\bar{5}_0$	3
$eE^5/\underline{5\rfloor}$ 2									4_0	4
E =	0.		3	1	8	1	0	1	0	5
										6

$(E^2 = \quad .\ 1_{\bar{1}}\ 0_{\bar{4}}\ \bar{5}_{\bar{1}}\ \bar{1}_2\ 3_{\bar{4}}$

where $e = 0.16722 = 0.02\bar{3}\bar{3}22$

Bring 0.3 into the answer.

Row 2: we multiply e and E : $CP\left[{2 \atop 3}\right] = 6 = 1_{\bar{4}}$.

Bring 1 into the answer.

Row 2: $CP\left[{2\bar{3} \atop 31}\right] = \bar{7},\ \bar{7} + \overline{40} = \bar{5}_3$

Bring down $\bar{2}$ into the answer

Row 2: $CP\left[{2\bar{3}\bar{3} \atop 312}\right] = \overline{16},\ \overline{16} + 30 = 2_{\bar{6}}$

Row 3: we find E^2 (row 6) then multiply rows 2, 6, divide by 6 and change the sign: $CP\left[{1 \atop 1}\right] = 1$, $CP\left[{\bar{1}\bar{5} \atop 10}\right] = \bar{1}5$ giving 15 = [illegible],

$5 \div 6 = 1\ r\bar{1}$, put $\bar{1}_{\bar{1}}$.

Bring down 1 into the answer.

Etc.

We get E = 0.3081010 to 7D.P.

Although M, and therefore E, take all values between 0 and 2 π, by using the angle to the nearest quadrant boundary we can always arrange that the quantity we calculate is less than about π/4. For example, if M = 3 radians, so that E = 3, we let E = π – A:

$$\therefore M = \pi - A - e\sin(\pi - A)$$
$$= \pi - A - e\sin A$$

from which we can find A, and then E since $E = \pi - A$

6.3 ILLUSTRATION 3

$$x + e^x = 2$$

$$\therefore x + 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \div \ldots.$$

$$\therefore x = 0.5 - \frac{x^2}{4} - \frac{x^3}{12} - \frac{x^4}{48} - \frac{x^5}{240} - \ldots.$$

0.5		0 . 5				1
$-x^2/4$	4		$\bar{5}_{5}$	$\overset{+}{1}_{\bar{6}}$	$0_{\bar{6}}$	2
$-x^3/12$	3		1	$3_{\bar{4}}$	$\bar{2}_{1}$	3
$-x^4/48$	4			$\bar{1}_{\bar{1}}$	$2_{\bar{3}}$	4
$-x^5/240$	5				1_{0}	5
x =		0 . 5	$\bar{6}$	3	$\bar{1}$	6

Bring down 0.5.

Row 2: D(5) = 25, 25 ÷ 4 = 5 r5, put $\bar{5}_5$

Row 3: we multiply rows 2,6 and divide by 3: CP[ ] $= \overline{25}, \overline{25} \div 3 = \bar{7} r \bar{4}$
put $\bar{1}3_{\bar{4}}$.

Bring down $\bar{6}$

Row 2: $D(5\bar{6}) = \overline{60}, \overline{60} + 50 = \overline{10}, \overline{10} \div 4 = \bar{1}$ r$\bar{6}$, put $1_{\bar{6}}$.

Row 4: we multiply rows 3,6 and divide by 4: $CP[\frac{\bar{1}}{5}] = \bar{5} \quad \bar{5} \div 4 = \bar{1}_{\bar{1}}$

Bring down 3 into the answer.

Etc.

TUTORIAL-6

CHAPTER-[illegible]

[illegible] Transcendental Equations

Solve the following transcedental equations.

1. $2x + 3\,\mathrm{Sin}\,x = 1$
2. $\mathrm{Sin}\,x = 1/2$
3. $\mathrm{Cos}\,x = -1/2$
4. $\mathrm{Sin}2x - 2x = 1/2$
5. $2\mathrm{Cos}x = \sqrt{3}/2$
6. $\mathrm{Cos}\,x - x = -1/2$
7. $\mathrm{Cosec}\,x = -2$
8. $\mathrm{Sin}\,x = \cos 3x$
9. $\tan x = -1$
10. $\mathrm{Sin}\,9x = \mathrm{Sin}\,x$
11. $\mathrm{Cot}\,x = 1$
12. $\mathrm{Sec}\,\theta = 2$
13. $\mathrm{Cosec}\theta = 2/\sqrt{3}$
14. $\mathrm{Sin}3x = \mathrm{Sin}2x$
15. $e^{x} + 2x = 4$
16. $3e^{x} - 5x = 2$
17. $\mathrm{Cos}mx = \mathrm{Cos}nx$
18. $e^{x} = 2$
19. $\mathrm{Cos}\,x - \mathrm{Cos}2x = \mathrm{Sin}\,x$
20. $\mathrm{Sin}\,x + \mathrm{Cos}\,x = \sqrt{2}$
21. $e^{x} + 3x^{2} = 5$

वैदिक गणित अध्ययन

CHAPTER-7
CALCULUS

REFERENCE: This chapter has been drawn from Chapter 16 of the book DISCOVER VEDIC MATHEMATICS by K.R. WILLIAMS published by Vedic Mathematics Research group, LONDON(U.K.).

7.1 INTRODUCTION Chalana Kalana

This is the sutra covering applications of the differential and integral calculus. It is well known that these applications are many and varied. The following examples illustrate some apparently unused techniques which simplify calculus work, and which make use of the Vedic sutras.

7.1.1 Example 1

Solve the equation $5x^2 - 3x - 2 = 0$

We use the relation that the first derivative is the square root of the discrimminant. The discrimminant is the square of the middle coefficient minus the product of twice the first coefficient and twice the last coefficient.

So we have $10x - 3 = \sqrt{(-3)^2 - 2.5.2.(-2)} = \sqrt{49} = \pm 7.$

$\therefore \quad x = 1 \text{ or } -\frac{2}{5}$

This simple and symmetrical formula replaces the notoriously complicated one usually used for solving quadratic equations.

7.1.2 Example 2

Locate the turning points of the graph of $y = \frac{x}{1 + x^2}$.

We differentiate numerator and denominator of this fraction to form a new fraction which we equate to the original one:-

$$\frac{x}{1 + x^2} = \frac{1}{2x} \qquad \ldots \qquad (1)$$

By cross-multiplication we then solve this equation- $2x^2 = 1 + x^2$

$\therefore \quad x = \pm 1$

To find y we can substitute these values of x into the simpler right-hand side of equation (1), and get $(1, \frac{1}{2})$ and $(-1, -\frac{1}{2})$ as the two turning points.

This problem would normally be solved by using the formula for differentiation of a quotient, equating the result to 0, and solving the equation.

7.1.3 Example 3

The sutra 'The first Derivative is the Sum of the Factors' refers to the relationships between the factors of a polynomial and its successive derivatives.

(a) $x^2 + 3x + 2 = (x+1)(x+2)$,
and the first derivative, $D_1 = 2x + 3 = (x+1)(x+2) = \Sigma A$
where ΣA denotes the sum of the factors.

(b) $x^3 + 6x^2 + 11x + 6 = (x+1)(x+2)(x+3)$
$D_1 = 3x^2 + 12x + 11 = (x+1)(x+2) + (x+2)(x+3) + (x+1)(x+3) = \Sigma AB$
where ΣAB denotes the sum of the products of all pairs of factors.
$D_2 = 6x + 12 = 2!(3x+6) = 2!(x+1+x+2+x+3) = 2!\Sigma A$.

(c) Similarly, for quartics we find $D_1 = \Sigma ABC$,
$D_2 = 2!\Sigma AB$,
$D_3 = 2!\Sigma A$

and so on.

7.1.4 Example 4

Using the 'Calculus sutra' with 'Transpose and Apply' we can find the sum of the squares, cubes etc. of the roots of a polynomial. If $f(x) = (x-a)(x-b)(x-c)\ldots(x-n)$ so that $a, b, c, \ldots n$ are the roots of $f(x)$, then using a result of the previous example:-

$$\frac{f'(x)}{f(x)} = \frac{\Sigma ABC\ldots(N-1)}{\Sigma ABC\ldots N} = \frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} + \ldots \frac{1}{x-n} \quad \ldots (2)$$

Now, using the 'Transpose and Apply' formula

$$\frac{1}{x-a} = \frac{1}{x} + \frac{a}{x^2} + \frac{a^2}{x^3} + \ldots.$$ with similar expressions for the other fractions in (2). Summing these expressions equation (2) becomes:-

$$\frac{f'(x)}{f(x)} = \frac{m}{x} + \frac{S_1}{x^2} + \frac{S_2}{x^3} + \frac{S_3}{x^4} + \ldots$$ for a polynomial with m roots,

where S_1, S_2, S_3 etc. are the sums of the 1st, 2nd, 3rd etc. powers of the roots.

This means that if we have a polynomial, $f(x)$, we can divide it into its first derivative, and by picking out the coefficient of the appropriate power of x, we can find the sum of whatever power of the roots we like.

Suppose, for example, we have the cubic equation -

$$x^3 + 3x^2 - 2x - 1 = 0$$

Then $f(x) = x^3 + 3x^2 - 2x - 1$, and $f'(x) = 3x^2 + 6x - 2$.
We then divide these by 'Transpose and Apply' :-

$$\begin{array}{rrr|rrrrrr}
x^3 + 3x^2 & -2x & -1\,) & 3x^2 & +6x & -2 & & & \\
-3 & +2 & +1 & & -9 & 6 & 3 & & \\
 & & & & & 9 & -6 & -3 & \\
 & & & & & & -39 & 26 & 13 \\
\hline
 & & & \multicolumn{6}{l}{\frac{3}{x} - \frac{3}{x^2} + \frac{13}{x^3} - \frac{42}{x^4} + \ldots\ldots} \\
\hline
\end{array}$$

From which we see $S_1 = -3$,
$S_2 = 13$,
$S_3 = -42$ and so on.

By dividing into the second derivative it is possible to extend this method to find the sum of products of pairs of roots and sums of other combinations of products of the roots.

7.1.5 Example 5

Successive derivatives can also be used for detecting repeated factors of polynomials. If a factor is repeated then it is also a factor of the first derivative. If the factor is repeated twice it is a factor of the first and second derivatives, and so on.

Factorise $E = x^3 - 4x^2 + 5x - 2$
$D_1 = 3x^2 - 8x + 5 = (3x - 5)(x-1)$
$D_2 = 6x - 8$

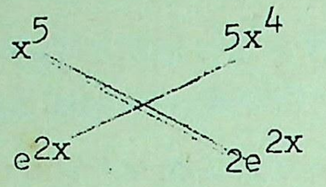
Therefore, since $(x-1)$ is a factor of E and D_1, $(x-1)$ is a factor repeated twice. So, using 'The First by the First and the Last by the Last', we get $E = (x-1)^2(x-2)$.

If, on the other hand, we know that a polynomial, E, has a repeated factor, we can find the H.C.F. of E and its first derivative, and thereby get the repeated factor.

7.1.6 Example 6

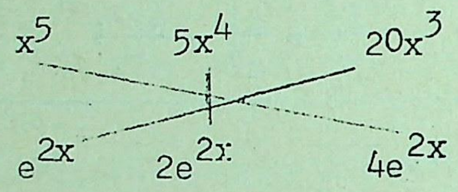
Find the 1st, 2nd and 3rd derivatives of $x^5 . e^{2x}$.
Here we make use of the 'Vertically and Cross-wise' formula

We write the two parts of the product, x^5 and e^{2x} one beneath the other, and put their derivatives next to them:-



Then by cross-multiplication $D_1 = x^5.2e^{2x} + 5x^4.e^{2x}$.

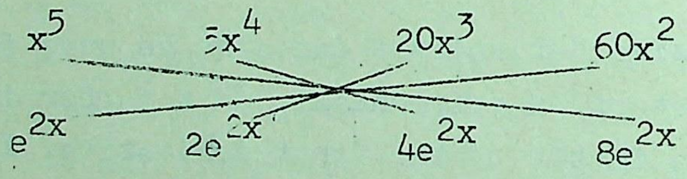
For the second derivative we put the first and second derivatives and multiply vertically and cross-wise:-



Then $D_2 = x^5.4e^{2x} + 2.5x^4.2e^{2x} + 20x^3.e^{2x}$.

The extra factor, 2, that comes into the middle term here is from Pascal's triangle; in fact the three terms are multiplied by 1, 2, 1 respectively.

For the third derivative we differentiate three times:-



The four products are multiplied by 1, 3, 3, 1 respectively

$$D_3 = x^5.8e^{2x} + 3.5x^4.4e^{2x} + 3.20x^3.2e^{2x} + 60x^2.e^{2x}.$$

This process is equivalent to Leibniz' theorem, which says that the nth derivative of a product u.v. is given by

$$(uv)_n = u_0v_n + \left[{n \atop 1}\right]u_1v_{n-1} + \left[{n \atop 2}\right] u_2v_{n-2} + \cdots\cdots$$

This gives us the product, of course, but does not bring out the underlying vertical and cross-wise pattern, which makes the Vedic method much more direct and practical.

There are similarities here with the 'Vertically and Cross-wise' method for finding the product of two numbers, and the corresponding method for the product of three numbers is similar to obtaining derivatives of products of three functions.

$$\therefore I = \frac{x^5}{5} \ln x - \int \frac{1}{x} \cdot \frac{x^5}{5} dx = \frac{x^5}{5} \ln x - \int \frac{x^4}{5} dx = \frac{x^5}{5} \ln x - \frac{x^5}{25} + c.$$

7.2.4 Example 10

$$\int \frac{x^2}{1 + x^3} = \frac{1}{3} \ln(1 + x^3) + c.$$

This integral can be found 'By Mere Observation'. We observe that, except for a numerical factor, the numerator is the derivative of the denominator, so that it is a ln function.

Finally we look at two examples of differential equations solved by means of the formula 'By the Completion or Non-completion'. We will use subscripts to denote derivatives, as before: so y_2 is the second derivative of y with respect to x.

7.2.5 Example 11

$$xy_1 + 2y = 3x$$

We recognise here that we would have a perfect first derivative on the LHS if we multiplied it by x:-

i.e. $x^2y_1 + 2xy = 3x^2$

$$\therefore (x^2y)_1 = 3x^2$$

$$\therefore x^2y = x^3 + c$$

7.3 DISCUSSION:

This beautiful written book by Prof K.R. WILLIAMS DISCOVER VEDIC MATHEMATICS in Second Edition also shows how Vedic Sutras can be used in solving examination problems at O and A level. The questions are taken from the University of LONDON school examination(the whole papers for June 1985). Both the multichoice paper and Paper-2 have been taken for each examination.

7.2 INTEGRATION:

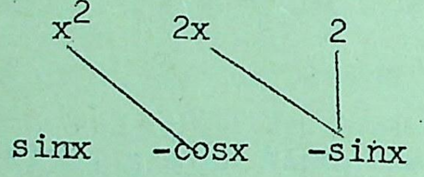
7.2.1 Example 7

$I = \int x^7 dx$ The integral of a simple function like this is obtained by applying the formula 'By One more than the One Before'. One more than 7 is 8, so the integral is $\frac{x^8}{8} + c$.

7.2.2 Example 8

$I = \int x^2 \sin x \, dx$

We write the two parts of the product one beneath the other, then differentiate one line and integrate the other:-

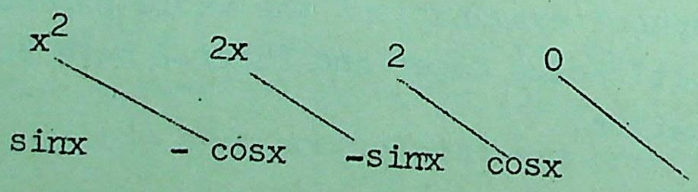


Then using 'Vertically and Cross-wise' we multiply cross-wise, as indicated until we reach a vertical product which we can integrate at sight. The signs of the products alternate +, −, +, −, etc.

$$\therefore I = -x^2 \cos x - (-2x\sin x) - \int 2\sin x dx$$
$$= -x^2 \cos x + 2x\sin x + 2\cos x + c$$

This integration would normally be accomplished by two applications of 'integration by parts'.

Alternatively, we can continue the differentiation and integration until a zero is reached:-

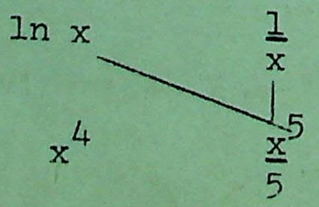


and just multiply cross-wise.

$$\therefore I = -x^2 \cos x - (-2x\sin x) + 2\cos x - 0 + c$$

7.2.3 Example 9

$I = \int x^4 \ln x dx$



वैदिक गणित अध्ययन

CHAPTER 8

EVALUATION OF DETERMINANTS

Reference: Vertically and Crosswise

8.1 Sum of Products

First Method Here we employ the sutra 'by alternate elimination and retention.'

8.1.1 Illustration 1 : Calculate 6 x 9 + 7 x 8 + 4 x 3

Each of the three products is a two-digit figure; eliminating the first figure in each case, we have 4 + 6 + 2 (the last figures of 54, 56 and 12). Hence the last digit is 2, with 1 to carry. Now we eliminate the last digits, giving: 1 (carried) + 5 + 5 + 1 = 12

Answer 122

8.1.2 Illustration 2: Calculate 7 x 9 + 6 x 4 + 3 x 8

Last digits give : 3 + 4 + 0 = 7

First digits give :6 + 2 + 4 = 12

Answer 127

8.1.3 Illustration 3: 6x 7 + 18 x 9 + 14 x 6 + 3 x 8

Here we observe that the four products in question are : 42, $2x9^2$ = 2 x 81 = 162, 2 x 7 x 6 = 2 x 42 = 84 and 24

Last digits give: 2+2+4+4 = 2 and 1 to carry

First digits give: 1 (carried) +4 + 16 + 8 + 2 = 31

Answer 312

Note that for each product the last digit is given by the product of the last digits. But here in the last example we are beginning to stray into the territory of the second method, which will now be considered.

8.2. Second Method This [illegible] on the 'vertically and crosswise' sutra.

8.2.1 Illustration 4: Calculate 43 x 72 + 54 x 31

We are going to carry out two 'vertically and crosswise' multiplications imultaneously. In the units place we have:

3 x 2 + 4 x 1 = 10

Write down 0, and carry 1

So far we have:

$$\begin{array}{ccc} 43 & & 54 \\ \times & + & \times \\ 72 & & 31 \\ \hline & & 0 \\ & & 1 \end{array}$$

Next, in the tens place we have:

1 + 4 x 2 + 3 x 7 + 5 x 1 + 4 x 3 = 47 .

Note the scope here for employing the first method. We now write down 7, and carry 4. Thus, so far we have written:

$$\begin{array}{cccc} 4 & 3 & 5 & 4 \\ \underline{7} & \underline{2} & \underline{3} & \underline{1} \\ & & 7 & 0 \\ & & 4 & 1 \end{array}$$

Next, for the hundreds place, we have:

4 (carried) + 4 x 7 + 5 x 3 = 47

Hence the answer is 4770.

A more convenient layout for the calculation is shown in the following example.

8.2.2 Illustration 5 Evaluate 213 x 426

+ 354 x 631

+ 652 x 172

$$\begin{array}{rccc} \times & 2 & 1 & 3 \\ & 4 & 2 & 6 \\ + & & & \\ \times & 3 & 5 & 4 \\ & 6 & 3 & 1 \\ + & & & \\ \times & 6 & 5 & 2 \\ & 1 & 7 & 2 \\ \hline \end{array}$$

$$4\ 2\ {}_{10}6\ {}_{12}2\ {}_{5}5\ {}_{2}6$$

The larger digits, such as the 7's, and 8's and 9's, can be avoided by using the vinculum.

8.3 Second Order Determinants

Methods for mental evaluation of second order determinants, and more generally of sums of products, make use of the preceding methods.

8.4 Third Order Determinants

A third order determinant is a layout, or matrix of elements specifying nine multiplications and various additions and subtractions, leading finally to a single figure. Thus a third order determinant is a formula reducing $3^2 = 9$ figures to one.

8.4.1 Illustration 6 :

Evaluate $D = \begin{vmatrix} 2 & 6 & 7 \\ 5 & 4 & 8 \\ 3 & 1 & 9 \end{vmatrix}$

Let us separate the first row from the second and third by a partition:

$$D = \left|\begin{array}{ccc} 2 & 6 & 7 \\ \hdashline 5 & 4 & 8 \\ 3 & 1 & 9 \end{array}\right|$$

Three second-order determinants arise from the second and third rows :

$$\begin{vmatrix} 5 & 4 \\ 3 & 1 \end{vmatrix} = 7 \; ; \; \begin{vmatrix} 5 & 8 \\ 3 & 9 \end{vmatrix} = 2 \quad ; \quad \begin{vmatrix} 4 & 8 \\ 1 & 9 \end{vmatrix} = 28$$

The sign of the second of these is reversed (by calculating 8 x 3-5x9 = -21), and the three results, 7, -21, and 28, are noted in a fourth row:

$$\begin{array}{ccc} 2 & 6 & 7 \\ \hdashline 5 & 4 & 8 \\ 3 & 1 & 9 \\ \hline -7 & -21 & 28 \end{array}$$

The first and fourth rows are now multiplied together 'vertically and cross-wise', i.e. we take

$$2 \times 28 + 6 \times (-21) + 7 \times (-7) = -119$$

$$\therefore \quad D = -119$$

8.4.2 Illustration 7: Evaluate $E = \begin{vmatrix} -3 & 7 & 1 \\ 2 & 4 & 3 \\ 5 & 8 & -4 \end{vmatrix}$

$$\therefore \quad E = \begin{array}{|ccc|} \multicolumn{1}{c}{-26} & \multicolumn{1}{c}{-11} & \multicolumn{1}{c}{+17} \\ \hline -3 & 7 & 1 \\ 2 & 4 & 3 \\ \hdashline 5 & 8 & -4 \end{array}$$

$$\underline{E = +277}$$

8.4.3 Illustration 8 : $F = \begin{array}{r|cc:c|} 37 & 7 & -3 & 9 \\ -32 & 3 & 4 & 5 \\ 19 & -1 & 5 & 7 \end{array}$

$$= \underline{270}$$

Note that the partitioning has been applied in various places; in Example 8 it is by columns. The multiplication pattern,

([illegible] Chapter 1.)

now becomes,

Since interchanging rows and columns does not change a determinant, evidently the sutra, 'vertically and cross-wise', needs to be understood as applying to both these patterns.

Note also that, of the three second-order determinants calculated after partitioning, that with negative sign contains elements spaced one row (or column) apart; adjacent elements lead to a positive sign. This observation is useful in dealing with fourth and higher order determinants. The following example should make the point clear.

8.5 Fourth Order Determinants

Partitioning centrally (either horizontally or vertically) we now proceed to evaluate second order determinants either side of the partition.

8.5.1 Illustration 9:

Evaluate $D = \begin{vmatrix} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{vmatrix}$

$$= \begin{array}{cccccc} + & - & + & + & - & + \\ -3 & -17 & -16 & -32 & -28 & 12 \\ \hline \multicolumn{6}{c}{\left| \begin{array}{cccc} 2 & 5 & 7 & 8 \\ 3 & 6 & 2 & 4 \\ \hline 3 & 5 & 7 & 6 \\ 1 & 6 & 9 & 7 \end{array} \right|} \\ \hline 13 & 20 & 15 & 3 & -1 & -5 \end{array}$$

The second order determinants above the partition are:

$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -3$, $\begin{vmatrix} 2 & 7 \\ 3 & 2 \end{vmatrix} = -17$; $\begin{vmatrix} 2 & 8 \\ 3 & 4 \end{vmatrix} = -16$

$\begin{vmatrix} 5 & 7 \\ 6 & 2 \end{vmatrix} = -32$; $\begin{vmatrix} 5 & 8 \\ 6 & 4 \end{vmatrix} = -28$; $\begin{vmatrix} 7 & 8 \\ 2 & 4 \end{vmatrix} = 12$

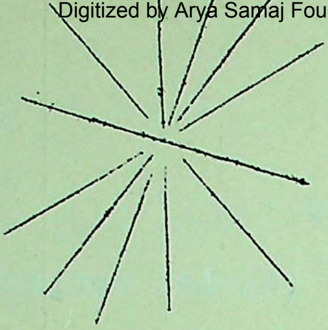
These are placed above the determinant in a row, the associated signs being placed above, '+' for adjacent columns '-' for those spaced one column apart, '+' for those spaced two apart.

The second-order determinants below the partition are obtained in the same sequence:

$\begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix} = 13$; $\begin{vmatrix} 3 & 7 \\ 1 & 9 \end{vmatrix} = 20$; $\begin{vmatrix} 3 & 6 \\ 1 & 7 \end{vmatrix} = 15$

$\begin{vmatrix} 5 & 7 \\ 6 & 9 \end{vmatrix} = 3$; $\begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix} = -1$; $\begin{vmatrix} 7 & 6 \\ 9 & 7 \end{vmatrix} = -5$

The two outermost rows of figures are now multiplied together according to the pattern,



associating the relevant signs with them. i.e. we have:

$$D = +(-3)(-5)-(-17)(-1)+(-16)(3)+ \ldots\ldots + (12)(13)$$

$$= \underline{186}$$

EXAMPLES

1. $D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 7 & 1 \\ 5 & 6 & 5 \end{vmatrix}$ (D = 54)

2. $D = \begin{vmatrix} 4 & 1 & 8 \\ 6 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix}$ (D = 10)

3. $D = \begin{vmatrix} 2 & 3 & 2 \\ 4 & 7 & 3 \\ 2 & 1 & 3 \end{vmatrix}$ (D = -2)

4. $D = \begin{vmatrix} 4 & 5 & 5 & 7 \\ 7 & 6 & 8 & 8 \\ 2 & 2 & 2 & 3 \\ 3 & 4 & 9 & 4 \end{vmatrix}$ (D = 9)

5. $D = \begin{vmatrix} 2 & 9 & 4 & 3 \\ 5 & 6 & 7 & 1 \\ 2 & 4 & 3 & 5 \\ 1 & 4 & 3 & 2 \end{vmatrix}$ (D = 145)

6. $D = \begin{vmatrix} 3 & 2 & 4 & 6 \\ 2 & 5 & 7 & 1 \\ 2 & 4 & 1 & 3 \\ 5 & 4 & 6 & 2 \end{vmatrix}$ (D = 414)

वैदिक गणित अध्ययन

CHAPTER 9

INVERSION OF MATRICES

Reference: Vertically and Cross Wise

9.1 INTRODUCTION

It is well-known that the inverse of a 2 x 2 matrix can be written straight down. The same procedure can be extended to allow inverses of third and fourth degree square matrices to be written straight down. Use of this is made in the following examples.

9.1.1 Illustration 1: Given $A = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$

$$\therefore A^{-1} = \frac{1}{(1.4-2.3)} \begin{vmatrix} 4 & -3 \\ -2 & 1 \end{vmatrix}$$

The following explanation of the steps applies also to higher order matrices.

Steps for determining the elements of A^{-1} from the elements of A

1. Along the leading diagonal (from top left to bottom right) replace each element by its cofactor. (The reader is reminded that the cofactor of the element in the ith row and jth column is obtained by crossing out the row and column containing the element, taking the determinant of what remains, and multiplying by $(-1)^{i+j}$).
2. Elsewhere replace each element by the cofactor of its mirror-image in the leading diagonal. E.g. the mirror-image of 2 is 3 (in matrix A above), and the cofactor of this 3 is -2.
3. Divide by the determinant of A, $|A|$

9.1.2 Illustration 2: Find the inverse of $B = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \\ 3 & 1 & 4 \end{vmatrix}$

Solution : $B^{-1} = \frac{-1}{35} \begin{vmatrix} 18 & -5 & -11 \\ -10 & -5 & 10 \\ -11 & 5 & -3 \end{vmatrix}$

STEPS

1. Replace the top left-hand digit by its cofactor,

$$\begin{vmatrix} 5 & 2 \\ 1 & 4 \end{vmatrix} = 18$$

2. Similarly, for other elements on the leading diagonal replace 5 by $\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = -5$, and 4 by $\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$

3. Other elements are replaced by the cofactors of their mirror-images in the leading diagonal. E.g., in the first column, the 4 is replaced by the cofactor of 2, viz - $\begin{vmatrix} 4 & 2 \\ 3 & 4 \end{vmatrix} = -10$, and the 3 is replaced by the cofactor of the top right-hand 3, viz $\begin{vmatrix} 4 & 5 \\ 3 & 1 \end{vmatrix} = -11$.

4. The divisor, $|B|$, is obtained from the first row of B the first column of B^{-1}. More generally, the ith row of B times the ith column of B, or vice-versa, yields $|B|$.

EXAMPLES

Evaluate the following

1. $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}\right)$

2. $M = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{14} \begin{bmatrix} 4 & -2 \\ -3 & 5 \end{bmatrix}\right)$

3. $M = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 5 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{27} \begin{bmatrix} 16 & 2 & -13 \\ -14 & 5 & 8 \\ 3 & -1 & 6 \end{bmatrix}\right)$

4. $M = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 2 \\ 1 & 7 & 2 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{-1}{18} \begin{bmatrix} -6 & 3 & 0 \\ 2 & 5 & -6 \\ -4 & -19 & 12 \end{bmatrix}\right)$

5. $M = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{-1}{5} \begin{bmatrix} 0 & 1 & -2 \\ 5 & -2 & -6 \\ -10 & 1 & 13 \end{bmatrix}\right)$

6. $M = \begin{bmatrix} 7 & 11 & 9 \\ 2 & 8 & 4 \\ 4 & 9 & 8 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{70} \begin{bmatrix} 28 & -7 & -28 \\ 0 & 20 & -10 \\ -14 & -19 & 34 \end{bmatrix}\right)$

7. $M = \begin{bmatrix} 3 & 4 & 2 & 1 \\ 4 & 1 & 2 & 3 \\ 4 & 3 & 1 & 2 \\ 3 & 2 & 1 & 4 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{40} \begin{bmatrix} -11 & 7 & 21 & -13 \\ 9 & -13 & 1 & 7 \\ 19 & 17 & -29 & -3 \\ -1 & -3 & -9 & 17 \end{bmatrix}\right)$

8. $M = \begin{bmatrix} 4 & 9 & 2 & 5 \\ 3 & 2 & 2 & 1 \\ 4 & 6 & 7 & 2 \\ 8 & 9 & 1 & 1 \end{bmatrix}$ $\quad \left(M^{-1} = \frac{1}{432} \begin{bmatrix} -23 & 216 & -57 & 13 \\ 13 & -216 & 51 & 49 \\ -24 & 0 & 62 & -24 \\ 91 & 216 & -75 & -89 \end{bmatrix}\right)$

वैदिक गणित अध्ययन

CHAPTER - 10

CHANGE OF ROOTS OF EQUATION

10.1 . INTRODUCTION: In this chapter we consider how to change a polynomial to obtain another whose roots are related to the original equation by being -

- अ. Opposite in sign.
- B. Reciprocals.
- C. Multiples or sub-multiples
- D. Increased or reduced by a given amount .

10.2 To obtain an equation whose roots are opposite in sign to those of a given equation.

We transpose the sign of every alternate term, beginning with the second:

10.2.1. Illustration 1 :-

1) $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ has roots 1, 2, 3, - 4.

$x^4 + 2x^3 - 13x^2 - 38x - 24 = 0$ has roots -1, -2, - 3, 4.

10.2.2. Illustration 2 :-

2) $2x^2 + 5x - 3 = 0$ has roots $\frac{1}{2}$, - 3.

$2x^2 - 5x - 3 = 0$ has roots $-\frac{1}{2}$, 3.

10.2.3. Proof If f (x) = 0 is a polynomial equation, and if -y = x, then f (-y) = 0 is satisfied by every root of f(x) = 0 with its sign transposed . f(-y) = 0 has roots opposite in sign to those of f(x) =0, and since successive terms in any polynomial alternate between odd and even powers of the unknown, substituting -y for x effectively changes the sign of alternate terms.

10.3. To obtain an equation whose roots are reciprocals of those in a given equation.

We reverse the order of the coefficients:

10.3.1 Illustration 3 :- $2x^3 + 3x^2 - 8x + 3 = 0$ has roots $\frac{1}{2}$, 1, -3.

the coefficients are 2,3, -8, 3,

$3x^3 - 8x^2 + 3x + 2 = 0$ has roots 2, 1, $-\frac{1}{3}$.

Illustration 4 :-

वेदात् सर्वं प्रसिद्ध्यति
वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत है।
Vedas are the Eternal Source of All Knowledge

10.3.2 $15x^2 - 13x + 2 = 0$ has roots $\frac{2}{3}$, $\frac{1}{5}$.

$2x^2 - 13x + 15 = 0$ has roots $\frac{3}{2}$

10.3.3. Proof If $f(x) = 0$ is a polynomial equation, and if $x = \frac{1}{y}$, then $f(\frac{1}{y}) = 0$ will have roots reciprocal to those of $f(x)=0$. But making the substitution $x=\frac{1}{y}$ will, on multiplying through by the appropriate power of y give a polynomial equation with reversed coefficients.

Eg. If $ax^2 + bx + c = 0$

$$\frac{a}{y^2} + \frac{b}{y} + c = 0 \quad \text{becomes } a + by + cy^2 = 0$$

$$\text{ie. } cy^2 + by + a = 0$$

10.4 To obtain an equation whose roots are all equal to those of a given equation multiplied by a given number.

This comes under the 'Proportionately' formula of Vedic mathematics.

We multiply the coefficients of the given equation by successive powers of the given number, starting with zero:

10.4.1 Illustration 5 : $x^2 + 2x - 15 = 0$ has roots 3, - 5.

Suppose we want an equation with double those roots, ie. 6,-10.

Multiply the coefficients by 2^0, 2^1, 2^2:

$$x^2 + 2.2x - 2^2.15 = 0$$

or $x^2 + 4x - 60 = 0$ is the required equation.

10.4.2. Illustration 6: $x^2 + 2x - 15 = 0$ has roots 3, - 5.

For an equation with roots 9, -15 ie. 3 times the original roots:

$$x^2 + 6x - 135 = 0.$$

For an equation with roots $1\frac{1}{2}$, $-2\frac{1}{2}$ ie. half the original roots:

$$x^2 + x - 3\frac{3}{4} = 0$$

or $4x^2 + 4x - 15 = 0$

10.4.3. Illustration 7 : $2x^3 - 3x^2 - 2x + 3 = 0$ has roots $1\frac{1}{2}$, -1, 1.

An equation with twice these roots is

$2x^3 - 6x^2 - 8x + 24 = 0$ ie. multiply by 1, 2, 4, 8.

$x^3 - 3x^2 - 4x + 12 = 0$

10.4.4. Illustration 8 : $2x^3 - 3x^2 - 2x + 3 = 0$ as above

We can in fact start with any term and multiply by powers of 2 to the ritht, and divide by powers of 2 to the left. Thus we can multiply this equation by $\frac{1}{2}$, 1, 2, 4 to double its roots:

$$x^3 - 3x^2 - 4x + 12 = 0.$$

10.4.5. Illustration : $6x^2 - x - 15 = 0$ find an equation with 3 times the roots.

Multiply by $\frac{1}{3}$, 1, 3

$$2x^2 - x - 45 = 0$$

10.4.6. Proof If $f(x) = 0$ is a polynomial equation, and if $y = xn$, then $f(\frac{y}{n}) = 0$ will have roots n times those of $f(x) = 0$. But if $f(\frac{y}{n}) = 0$ then $y^p . f(\frac{y}{n}) = 0$ where p is the order of the polynomial.

Eg. If $ax^2 + bx + c = 0$

$$\frac{ay^2}{n^2} + \frac{by}{n} + c = 0$$

or $ay^2 + nby + n^2c = 0$.

CHAPTER - 11

THE SOLUTION OF LINEAR AND NON-LINEAR DIFFERENTIAL EQUATIONS

11.1. INTRODUCTION :

The method of series solution of differential equations is well-known. Bringing to bear the vedic mathematical system of sutras of Sri Bharati Krishna Tirthaji makes this a very powerful approach, as is shown by the following examples.

The procedure is applicable to a wide range of problems, in particular those where boundary conditions are given at the origin or at a single point.

We start with a simple linear example to illustrate the method. Except perhaps for the lay-out the procedure of the first example is familiar to western mathematicians.

11.1.1. Illustration 1.

Solve $2y + 3y' = 18 + 8x$ ....(1)

given $y'(0) = 4$

Here, and throughout this Chapter, $y' = \frac{dy}{dx}$

and $y'(0) = \frac{dy}{dx}$ evaluated with $x = 0$

An explanation of the steps will be given shortly, but the complete working and solution appears finally as follows:-

$$y \quad \overset{3}{a} + \overset{4}{bx} + \overset{0}{cx^2} + \overset{0}{dx^3} + \overset{0}{e\,x^4} + \ldots\ldots\ldots.(2)$$

$$y' = \underset{4}{b} + \underset{0}{2cx} + \underset{0}{3dx^2} + \underset{0}{4ex^3} + \underset{0}{5fx^4} + \ldots\ldots\ldots.(3)$$

i.e., the solution is, $\underline{y = 3 + 4x}$ exactly

Explanation

By Maclurin's Theorem, a series expansion can be used for y, as in Equation (2). Differentiating gives Equation (3), successive coefficients of x being arranged in successive columns. Purely for purposes of explanation we show the following layout:-

$$\begin{array}{llllllll} \text{Coefficients} & & & & & \text{Column} & & \\ & 1 & 1 & 2 & 3 & 4 & & \\ \hline 2 & y & = a \ + & bx \ + & cx^2 \ + & dx^3 & + \ldots\ldots\ldots & \text{Row 1} \\ 3 & y' & = b \ + & 2cx \ + & 3dx^2 \ + & 4ex^3 & + \ldots\ldots\ldots & \text{Row 2} \\ & & & 18 \ + & 8x & & & \text{Row 3} \end{array}$$

The left-hand side of Equation (1) translates into 2x Row 1 + 3x Row 2, and the coefficients '2' and '3' have here been placed in a column on the left as a reminder. To facilitate the working, the right-hand side terms of Equation (1) can be placed in a third row. Thus for each successive column we have,

$$2 \times \text{Row } 1 + 3 \times \text{Row } 2 = \text{Row } 3,$$

in accordance with Equation (1), And each successive column yields the next of the unknown, a, b, c, d, ...

The sutra here is, 'by alternate elimination and retemtion'

Since $y'(0) = 4$, $b = 4$, from Row 2

Write this in place, as shown in the complete solution.

To find 'a', Column 1 (absolute terms only) gives us,

$$2A + 3b = 18$$

Since b is known to equal 4, the mental procedure is, reduce 18 by 3 x 4, giving 6, and divide by 2 to obtain a = 3. Now write '3' above 'a' in Row 1.

So far we have:

$$y = \overset{3}{a} + \overset{4}{b}x + cx^2 + \ldots\ldots\ldots\ldots$$
$$y' = \underset{4}{b} + 2cx + 3dx^2 + \ldots\ldots\ldots\ldots$$

Similarly , in Column 2,

$$2b + 3 \times 2c = 8$$
$$2c = 8 - 2 \times 4 = 0$$
$$c = 0, \text{ etc.}$$

The same procedure shows that all subsequent terms are zero, and

$$y = 3 + 4x \text{ exactly,}$$

The next example introduces a non-linearity, namely the term y^2.

11.1.2. Illustration 2 :

Solve $y'' = y^2 + 1 + 2x^2 + x^4$..............(1)

Where $y(0) = 0$, and $y'(0) = 0$

The working can be written down as follows:

$$\begin{array}{lcccccccccc} \text{Column} & \text{I} & \text{II} & \text{III} & \text{IV} & \text{V} & \text{VI} & \text{VII} & \text{VIII} & \text{IX} & \text{X} \end{array}$$

$$y = \overset{0}{a} + \overset{0}{b}x + \overset{\frac{1}{2}}{c}x^2 + \overset{0}{d}x^3 + \overset{\frac{1}{6}}{e}x^4 + \overset{0}{f}x^5 + \overset{\frac{1}{40}}{g}x^6 + \overset{0}{h}x^7 + \overset{\frac{1}{336}}{i}x^8 + \overset{0}{j}x^9 + \quad (2)$$

$$y'' = \overset{1}{2c} + \overset{0}{6dx} + \overset{2}{12ex^2} + \overset{0}{20fx^3} + \overset{\frac{3}{4}}{30gx^4} + \overset{0}{42hx^5} + \overset{\frac{1}{6}}{56ix^6} + \overset{0}{72jx^7} + .. \quad (3)$$

Explanation

From the boundary conditions we have a = 0 and b = 0 As before, we work with coefficients of x, in succession , each step yielding another of the values of b, c, d, ...
To conform with the left-hand side of Equation (1), the successive terms for y^2 are given by successive duplexes (or dwandwa-yogas)+ of Equation (2)

For 'c'

From Column 1,

$$2c + a^2 = 1,$$

. Since we are eliminating all except the absolute terms. And since $a^2 = 0$, the mental procedure is, reduce 1 by 0 to obtain 1 as the value of 2 c. Hence we put 2c = 1 and c = ½, as shown in the complete solution.

For 'd'

We now eliminate all terms from Equation (1) except for the coefficients of x. The dwandwa-yoga of y^2 here is $2ab = 2 \times 0 \times 0$, and from Column II we have,

$$2ab + 6d = 0$$

$$d = 0$$

For 'e' (Column III)

The dwandwa-yoga is now $2ac + b^2$, and we have

$$2 \times 0 \times \tfrac{1}{2} + 0^2 + 12e = 2$$

we write 2 above 12e, and 1/6 above e.

For 'f' (Column IV)

Since the coefficient of x^3 in Equation (1) is zero, the term in y'' equals minus the dwanda-yoga term, i . e. $-2ad-2bc$, which is zero; hence f = 0. And so the solution continues. Had y (0) not been zero there would have been two solutions. As it is, the two solutions coincide.

The next example has two solutions, both of which are given. The boundary conditions are given at x = 1, and so a series expansion in powers of (x - 1) is used.

11.1.3. Illustration 3 :

Solve $2y + (y')^2 + 3y'' = 2\exp(x) + \exp(2x)$...... (1)

given $y(0) = 2$, $y'(0) = 1$

Here the presence of $(y')^2$ might lead us to expect two solutions, but being given $y'(0) = 1$, there is only one solution. We first expand the right-hand side of Equation (1) as a power series in x :

$$y + (y')^2 + 3y'' = 3 + 4x + 3x^2 + \frac{10}{3!}x^3 + \frac{18}{4!}x^4 + \frac{34}{5!}x^5 + \frac{66}{6!}x^6 + \frac{130}{7!} + \ldots$$

$$\text{Let } y = \overset{2}{A} + \overset{1}{B}x + \overset{0}{C}x^2 + \overset{\frac{1}{9}}{D}x^3 + \overset{\frac{7}{108}}{E}x^4 + Fx^5 + Gx^6 + Hx + \ldots\ldots.$$

$$y' = \overset{1}{B} + 2\overset{0}{C}x + 3\overset{\frac{1}{3}}{D}x^2 + 4\overset{\frac{7}{24}\,=\,0.3210\bar{1}}{E}x^3 + 5Fx^4 + 6Gx^5 + 7Hx^6 + 81x^7 + \ldots.$$

$$y'' = 2\overset{0}{C} + 6\overset{\frac{2}{3}}{D}x + 12\overset{\frac{7}{9}}{E}x^2 + 20Fx^3 + 30Gx^4 + 42Hx + 561x^6 + \ldots$$

I.e. the solution is :

$$\underline{y = 2 + x + 0.ix^3 + 0.064814815x^4 + \ldots\ldots\ldots}$$

11.1.4. Illustration 4 :-

Solve $y^{\frac{1}{2}}y' + (y')^2 = 6(2+x)^2$.......... (1)

given $y(0) = 4$

We make use of the vedic method for finding the square root of a polynomial or power series. This is essentially the same as that for finding the square root of a number in decimal form -

Initially we assume a power series for y, and allow the successive terms of $y^{\frac{1}{2}}$ to unfold above it, the first derivative being placed below. Since $y(0) = 4$, $a = 4$ and the first term for $y^{\frac{1}{2}}$ is $+ 2$ (or $- 2$, leading to another solution).

The solution proceeds :

$$y^{\frac{1}{2}} = +2 + \ldots\ldots..$$

$$y = \overset{4}{a} + bx + cx^2 + dx^3 + ex^4 + fx^5 + \ldots\ldots..$$

$$y' = b + 2cx + 3dx^2 + 4ex^3 + 5fx^4 + 6gx^5 + \ldots\ldots..$$

Now Equation (1) can be rewritten,

$y'(y^{\frac{1}{2}} + y') = 24 + 24x + 6x^2$............... (2)

Hence, proceeding vertically and crosswise, the coefficient of x^2 comes to $6(3d) + 3 \times 2 + 4 \times (3d) = 6$, where the right-hand-side '6' is the coefficient of x^2 on the right in Equation (2). Hence $d = 0$.

Continuing in this way we find that

$$y = 4 + 4x + x^2$$

$$= (2 + x)^2 \text{ exactly.}$$

This is one of four solutions.

11.2. Transcendental Equations

A special case of equations such as those presented so far is one containing no differentials, but simple terms in y, and functions of y, and functions of x. This is a type of transcendental equation, differing from those of Chapter 11 of vertical & crosswise in having two variables, one being considered to be dependent upon the other.

11.2.1. Illustration 5:-

$$\text{Solve} \quad y - \ln y = 1 - x^2, \qquad (1)$$

$$\text{Where} \quad y = y(x)$$

Assume that

$$y = a + bx + cx^2 + dx^3 + ex^4 + \ldots\ldots\ldots\ldots \quad (2)$$

Substituting in Equation (1) and differentiating w.r.t. x :

$$\frac{\overset{0}{b} + \overset{-1}{2c}x + \overset{0}{3d}x^2 + \overset{\frac{1}{4}}{4e}x^3 + \overset{0}{5f}x^4 + \overset{\frac{1}{32}}{6g}x^5}{\underset{1}{a} + \underset{0}{b}x + \underset{-\frac{1}{2}}{c}x^2 + \underset{0}{d}x^3 + \underset{\frac{1}{16}}{e}x^4 + \underset{0}{f}x^5 + \underset{192}{g}x^6}$$

$$= \underset{0}{-b} - (2 + \underset{-1}{2c})x - \underset{0}{3d}x^2 - \underset{-\frac{1}{4}}{4e}x^3 - \underset{0}{5f}x^4 - \underset{\frac{1}{32}}{6g}x^5 - \ldots\ldots (3)$$

Considering only absolute terms, we have,

$$b(2+b) = 24$$

i.e. we seek two numbers differing by 2, whose product is 24. clearly 4 & 6 or 4 & -6 fit the bill (hence a doubling up of the number of solutions). Using 4 & 6, we have b = 4.

The solution continues:

$y^{\frac{1}{2}} = 2 + 1x + \ldots\ldots\ldots$ Row 1

$y = \overset{4}{a} + \overset{4}{b}x + \ldots\ldots\ldots$ Row 2

$y' = \overset{4}{b} + \overset{2}{2}\,cx + 3dx^2 + \ldots$ Row 3

We are now ready to consider the coefficient of x, in Equation (2). The left-hand side of Equation (2) is satisfied by (mentally) adding Row 3 to Row 1, and multiplying cross-wise by Row 3. i. e.

$$y^{\frac{1}{2}} + y' = 6 + (1 + 2c)x$$

$$y' = 4 + 2cx + \ldots\ldots\ldots$$

the coefficient of x is (1 +2c) x 4 + 6 x 2c. And this must equal 24 (from the right-hand side of Equation (2).

$$2c = (24 - 3\text{x}4) \div 6 = 2,$$

$$\underline{c = 1}$$

Thus the solution so far is

$$y^{\frac{1}{2}} = 2 + 1x + 0x^2 + \ldots\ldots$$

$$y = \overset{4}{a} + \overset{4}{b}x + \overset{1}{c}x^2 + \ldots\ldots$$

$$y' = \overset{4}{b} + \overset{2}{2c}x + 3dx^2 + \ldots\ldots$$

For on continuing the square-rooting process, the next coefficient (that of x^2) is zero.

Considering now the coefficients of x^2 in Equation (2) we have,

$y^{\frac{1}{2}} + y'$ gives $6 + 3x + 3dx^2$

& y' gives $4 + 2x + 3dx^2$

STEPS		Equation Used
By observation,	$y(0) = 1$ is a solution *	(1)
	$a = 1$	(2)
Equate coefficients of unity:	$b = 0$	(3)
" " " x	: $4c = -2$	(3)
" " " x^2	: $3d = 0$	(3)
" " " x^3	: $8e = \frac{1}{2}$	(3)
" " " x^4	: $10f = 0$	(3)
" " " x^5	: $12g = \frac{1}{16}$	(3)

A solution is :

$$y = 1 - \tfrac{1}{2}x^2 + \frac{1}{16}x^4 + \frac{1}{192}x^6 + \ldots\ldots\ldots\ldots$$

---

* This is the only real solution of $y - \ln y = 1$

1. $y' + 2y + b = 4x + 16$ $\quad(y = 4 + 2x)$
2. $y' + (\frac{dy}{dx})^2 = 676x^4 + 624x^3 + 378x^2 + 108x + 20$ $\quad(y=2+4x+6x^2+13x^3)$
3. $y'' + y' - y = -10x^4 + 38x^3 + 126x^2 + 8x + 4$ $\quad(y=4x+2x^3+10x^4)$
4. $y' -3y'' + 6 = 4 - 14x -32x' - 24x^3$ $\quad(y = (1+2x)^3)$
5. $y'' + 2y' + 6y = 14x + 4x$ $\quad(y = 1 + 4x)$
6. $2y + 3y' + 6y^2 = 5\exp(x) + 6\exp(2x)$ $\quad(y = \exp(x))$
7. $4y' + 7y + 2 = 2 + 23\exp 4x - 11\exp x$ $\quad(y = \exp(4x)-\exp(x))$
8. $\int_0^x y\,dx + 3\int_0^x y\,dx = 16 + 6x^2$ $\quad(y = 4 + 3x)$
9. $\int_0^x\int_0^x y^2\,dx + \int_0^x y\,dx = 2x + \frac{7}{2}x^2 + 2x^3 + \frac{3}{4}x^4$ $\quad(y = 3x + 2)$

Tutorial 2

Use Anurupyana(अनुरूप्येण) and Adyamadyanantya-Mantyena (आद्यमाद्येनान्त्यमन्त्यैम) to solve the following Quadratic.

1. $x^2 + 12x + 36$
2. $3x^2 - 17x - 28$
3. $3x^2 - 1$
4. $3x^2 + xy - 14y^2$
5. $-2x^2 + 6x + 80$
6. $4x^2 + 10x - 6$
7. $2x^2 + 13xy + 6y^2$
8. $\frac{4}{9}x^2 - 25y^2$
9. $9x^2 + 4xy - 5y^2$
10. $x^2 + \frac{7}{10}x - 5/6$
11. $3x^2 + 19x + 6$
12. $7x^2 - 6xy - y^2$
13. $x^2 - 5/3\,x + 25/36$
14. $2x^2 - x - 15$
15. $6x^2 - 33x + 17$
16. $15x^2 - 17xy - 4y^2$
17. $36x^2 - 36xy + 9y^2$
18. $x^2 + 13x + 36$
19. $2x^2 + 11x + 9$
20. $4x^2 - 16y^2$

21 Check the answers of the above problems by Vedic method.

TUTORIAL-5

Quadratic and Cubic equations

Solve the following equations:-

1. $x^2 - 3x + 1 = 0$
2. $2x^2 - 5x + 7 = 0$
3. $x^2 + 8x - 6 = 0$
4. $x^2 - 2x - 1 = 0$
5. $2x^2 + 7x + 2 = 0$
6. $x^3 + 2x^2 - 5x + 2 = 0$
7. $x^3 + 4x^2 + 2x + 1 = 0$
8. $5x^2 - 10x + 7 = 0$
9. $4x^3 - 2x^2 + x + 1 = 0$
10. $2x^3 + 5x^2 + 6x - 64 = 0$
11. $8x^2 - 12x - 6 = 0$
12. $7x^2 - 50x + 8 = 0$
13. $3x^2 - 5x + 8 = 0$
14. $x^3 - 7x^2 + 5x + 2 = 0$
15. $x^2 + 4x + 8 = 0$

5980

पुस्तकालय
गुरुकुल कांगड़ी विश्वविद्यालय

GURUKUL
Accession 6-5-09
Class on
Cat on
Tag etc
Checked
Any Other 14/2/09

B
14-2-09

R
21/2/09

EAR

18-2-91

18.2.91

आध्यात्मिक अध्ययन संस्था, रुड़की (आऽस)

SPIRITUAL STUDY GROUP (SSG)

भारतवष का पुनरुत्थान होगा, पर वह शारीरिक शक्ति से नही, वरन् आत्मा की शक्ति द्वारा । वह उत्थान विनाश को ध्वजा लेकर नहीं, वरन् शाँति और प्रेम की ध्वजा से । मैं अपने सामने यह एक सजीव दृश्य अवश्य देख रहा हूं कि हमारा यह वृद्ध माता पुनः एक बार जागृत होकर अपने सिंहासन पर नवयौवनपूर्ण और पूर्व की अपेक्षा अधिक महामहिमान्वित होकर विराजमान है । शांति और आशीर्वाद के वचनों के साथ सारे ससार में उनके नाम का घोषणा कर दो ।

तुम तो ईश्वर की संतान हो अमर आनन्द के भागी हो, पवित्र और पूर्ण आत्मा हो । अतएव तुम कैसे अपने को जबरदस्ती दूबल कहते हो ? उठो, साहसी बनो, वीर्यवान होओ ।

केवल आध्यात्मिक ज्ञान ही ऐसा है जो हमारे दुःखों को सदा के लिए नष्ट कर दे सकता है।

—स्वामी विवेकानन्द

With a view to creat an opportunity for all on the university campus and the city, to know about our rich cultural and spiritual heritage and its validity and significance in the modern scientific age, a spiritual study group (SSG) has been formed.

1. Study, teaching, research and publications in Vedic Mathematics and Ancient Sciences.
2. Lectures, demonstrations and films about the Spiritual Science and meditation.
3. Visits to the centers of Spiritual Sciences.
4. Spiritual and devotional meetings.
5. Thought bulletin to spread elementry spiritual science.

Whether you are a child, or a student or a house wife or an old man, SSG has a scheme to offer to you which can make you enjoy peace,develop poise and there by help in your over all growth.

ANCIENT-VEDIC MATHEMATICS (A Magic Till Understood)

Salient Features—A complete System. A boon for all *competitions*. With a firm scientific footing. A revolution in concept of learning. Revealing mysterious qualities of numbers. Simple short, mathematical processes often with magic speed. Requires no prerequisite courses. Very frequently supersedes calculators. 2 hours/day for 11 months of it equals 12 years of normal course in maths.

Two Seminars and one workshop on Vedic Maths have been organised.

Ancient Vedic Maths-Pushp mala is being published.

Few opinions about Ancient Vedic Mathematics :—

The essence of Vedic Maths System is that it is simple, direct, one-line and mental. The Vedic sutras are easy to remember, easy to understand and a delight to use. The variety and simplicity of Vedic methods brings fun and amusement.

by K R. Williams (U.K.,

—"It is tempting to think of them (Indians) as computer scientists....................Let us not forge that among the great accomplishments of the Indian thinkers were the invention of zero, and of the Binary number system a thousand years before the West".

Dr. R. Briggs of NASA, California,
in Jl. of Artificial Intelligence, Spring, (85)

– Increases devotion towards our INDIA and its ancient Culture.

—If Vedic Maths was taught in every school, Computers would have developed in India.

S. S. G. organises Courses, Correspondence courses, Demonstration lectures all over INDIA on Vedic Mathematics. It also publishes and distributes 16 books on Vedic Maths.

For any other details kindly contact :

Dr. Narinder Puri

Reader, Civil Engg Deptt. University of Roorkee, 41/2 Amod Kunj, UOR Roorkee-247667 (UP) Ind